Ewkons, a quasidistinguishable dark energy?

Pablo D. Sisterna

Departamento de Fisica
F.C.E.y N.
Universidad Nacional de Mar del Plata

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Outline of ideas and results

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- Dark energy is an elusive concept whose nature is far from being understood, both theoretically and observationally.
- Quasi distinguishable particles called "ewkons" obey unorthodox statistics, and have a negative relation between pressure and energy density.
- One can formulate an effective scalar field description of the ewkon fluid, obtaining cosmological solutions for the dark energy dominated epoch (can be considered as a one-parameter class of dark energy models).

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- Hoyuelos, Phys.Rev.E (Hoyuelos 2022) derived transition rates between levels with energy ϵ_1 and ϵ_2, with n_1 and n_2 particles respectively, in terms of the residual chemical potential.
- It was also shown that, if the transition rate depends on the number of particles at the destination level, then Fermi-Dirac (FD $+$) and Bose-Einstein (BE $-$) statistics are deduced.

Time reversed transitions and alternative statistics

- In a time reversed scenario, the transition rate depends on the number of particles in the initial level. Hoyuelos & P.S., Phys.Rev.E (Hoyuelos-Sisterna 2016), obtained ewkons $(+)$ and genkons $(-)$ statistics, with occupation numbers $\bar{n}_\pm = e^{-(\epsilon-\mu)/T} \pm 1$.

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- An ideal gas of ewkons has negative pressure; barotropic parameter even close to $-1 \rightarrow$ possible dark energy?
- (Hoyuelos-Sisterna 2016) derived Ewkon statistics from the assumption of free diffusion in energy space and the adjustment of an "interpolation parameter".

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- Hoyuelos, J. Stat. Mech.: Theory Exp. (Hoyuelos-2018b) analyzed a massless scalar field of ewkons.

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- Assuming that the present density of ewkons accounts for the bulk of the present density of dark energy, we will check the consistency of the hypothesis that dark energy has the statistics of ewkons.

Quantum (alternative) statistics: grand partition function

Consider the grand partition function of a system of non-interacting particles in contact with a reservoir at temperature T and chemical potential μ: $\mathcal{Z} = \prod_{\mathbf{k}} \mathcal{Z}_{\mathbf{k}}$, where **k** refers to the mode with wave vector **k** and, in the number eigenstates base.

$$\mathcal{Z}_{\mathbf{k}} = \sum_{n} \delta_n \, e^{-n(\epsilon_{\mathbf{k}} - \mu)/T} \tag{1}$$

is the grand partition function for particles that have energy $\epsilon_{\mathbf{k}}$; δ_n is the statistical weight factor.

Statistical weight factors

- Bose-Einstein (BE) statistics $\rightarrow \delta_n = 1 \ \forall n$
- Fermi-Dirac (FD) statistics $\rightarrow \delta_0 = \delta_1 = 1$ and $\delta_n = 0$ for $n \geq 2$.
- Maxwell-Boltzmann statistics $\rightarrow \delta_n = 1/n!$.
- To calculate the grand partition function $\mathcal{Z}_{\mathbf{k}}$, the vacuum energy term $\epsilon_{\mathbf{k}}/2$ is removed as usual, if we do not want the (normal matter) vacuum to exert any pressure.

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- A priori, "Identicality" and "indistinguishability" refer to different concepts.
- The spin-statistics theorem applies only to indistinguishable particles.
- For example, two electrons with opposite spin can be treated as approximately distinguishable.
- In principle, quantum mechanics can be developed without the symmetrization postulate (that, in turn, implies the indistinguishability postulate), allowing more general statistics.

Late acceleration of the Universe suggests Cosmological Constant/Dark Energy

- Early Dark Energy models (e.g. Ultra-Light dissipative Axions, Berghaus Karwal, 2020).
- Graduated Dark Energy (Akarsu, Barrow, Escamilla Vazquez, 2020).
- Late Dark Energy models (e.g. wCDM models, Di Valentino, 2017).
- Dynamical dark energy parameterizations with one or two free parameters (e.g. Yang, Pan, Di Valentino, Saridakis Chakraborty, 2019).
- Holographic Dark Energy (Li, 2004).

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- Phantom Braneworld Dark Energy (Alam, Bag Sahni, 2017).
- Phantom Dynamical Dark Energy (Dahmani, Bouali, El Bojaddaini, Errahmani T. Ouali, 2023).
- Chameleon dark energy (J. Khoury, 2013).
- Interacting Dark Energy (L. Amendola, 2000), etc.

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- We consider ewkons as a candidate for Late Dark Energy Models, to be justified below.

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- The density and pressure of an ideal gas of ewkons obtained for large and small temperature regimes.

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- Functional form of the effective potential for the dark energy dominated era.
- Dynamics of the scalar field.
- Conclusions and comparison with present models of dark energy.

Partition Function, density and pressure of ewkons

The lowest energy state for ewkons is $|1\rangle$ (not $|0\rangle$ as for bosons):

$$\mathcal{Z}_{\mathbf{k}} = \sum_{n=1}^{\infty} \delta_n e^{-n(\epsilon_{\mathbf{k}}-\mu)/T}$$

$$= e^{-(\epsilon_{\mathbf{k}}-\mu)/T} \sum_{n'=0}^{\infty} \delta_{n'+1} e^{-n'(\epsilon_{\mathbf{k}}-\mu)/T} \qquad \text{(n=n'+1)}.$$

We define the statistical weight, $\delta_{n'+1} \equiv 1/n'!$, as the Gibbs factor for distinguishable particles (Hoyuelos-2018a). Then

$$\mathcal{Z}_{\mathbf{k}} = \exp\left[-(\epsilon_{\mathbf{k}}-\mu)/T + e^{-(\epsilon_{\mathbf{k}}-\mu)/T}\right].$$

Mean occupation number

Therefore

$$\bar{n}_{\mathbf{k}} = T\frac{\partial \ln \mathcal{Z}_{\mathbf{k}}}{\partial \mu} = e^{-(\epsilon_{\mathbf{k}}-\mu)/T} + 1.$$

Main ingredients of the ewkon scenario leading to this equation:

- Lowest energy state other than the vacuum.

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Main motivations:

- This number statistics can be deduced from simple assumptions on the transition rates (Hoyuelos-2022).

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Main motivations:

- This number statistics can be deduced from simple assumptions on the transition rates (Hoyuelos-2022).
- The resulting thermodynamic properties connect with dark energy.

Thermodynamic properties of an ideal gas of (nearly) massless ewkons

The non-relativistic case was analyzed in Hoyuelos-2018a, where an upper bound for the mass of 0.006 eV was obtained, suggesting relativistic Ewkons, so we assume $\epsilon_{\mathbf{k}} = \sqrt{m^2 + k^2} \simeq k$, and zero chemical potential. The total grand partition function is

$$\begin{aligned} \frac{1}{V} \ln \mathcal{Z} &= \frac{1}{(2\pi)^3} \int \mathrm{d}\mathbf{k}\, g \ln \mathcal{Z}_{\mathbf{k}} \\ &= \frac{1}{2\pi^2} \int_0^{\epsilon_{\mathrm{m}}} \mathrm{d}\epsilon\, g \epsilon^2 (e^{-\epsilon/T} - \epsilon/T) \end{aligned}$$

where g is the degeneracy, and we consider $1 < g < 10$ (sub-index $\mathbf{k}$ was removed in $\epsilon_{\mathbf{k}}$ for simplicity).

Density and pressure

We introduce a maximum energy $\epsilon_{\rm m}$ to avoid divergences (to be fixed later using the energy conservation equation). The energy density and pressure are:

$$\begin{aligned} \rho &= \frac{g}{2\pi^2}\int_0^{\epsilon_{\rm m}} {\rm d}\epsilon\,\epsilon^3\,\bar{n}_{\mathbf{k}} \qquad (2)\\ &= \frac{gT^4}{8\pi^2}\left[(u^4+24)e^u - 4u^3 - 12u^2 - 24u - 24\right]e^{-u}, \\ p &= \frac{T}{V}\ln\mathcal{Z} = -\frac{gT^4}{8\pi^2}\left[(u^4-8)e^u + 4u^2 + 8u + 8\right]e^{-u}, \qquad (3)\end{aligned}$$

with $u = \epsilon_{\rm m}/T$. The equation of state or barotropic parameter is

$$w = \frac{p}{\rho} = -\frac{(u^4-8)e^u + 4u^2 + 8u + 8}{(u^4+24)e^u - 4u^3 - 12u^2 - 24u - 24}. \qquad (4)$$

Some references of dark energy parametrization

Although this last expression is a quotient of quasi polynomials in u, it does not resemble any known dark energy parametrization, such as

- Sendra-Lazkoz, 2012,
- Feng-Shen-Li-Li, 2012,
- Barboza-Alcaniz, 2008,
- Chevallier-Polarski, 2001 & Linder, 2003,
- Jassal-Bagla-Padmanabhan, 2005,
- Models with a Chaplygin like fluid (Shenavar-Javidan, 2020 & Bento-Bertolami-Sen, 2004).

Ewkon energy conservation

The momentum of each ewkon particle decays as a^{-1} (a is the scale factor of the Universe), so $T \propto a^{-1}$ ($a_0 \equiv a(presenttime) = 1$, so $T = T_0/a$.

- We consider a universe in which dark energy, with density $\rho_{\rm de}$ and pressure $p_{\rm de}$, behaves as ewkons.
- We assume that there is no interaction with matter or radiation, so the energy conservation equation is

$$\dot{\rho}_{\rm de} = -3\frac{\dot{a}}{a}(\rho_{\rm de} + p_{\rm de}) = -3\frac{\dot{a}}{a}(w+1)\rho_{\rm de}. \qquad (5)$$

- Interactions may have been present during the very early stages of the universe, so we expect T_0 similar to T_0(CMB)$=$ 2.72548 K (Fixsen, 2009), or 2.34863 10^{-4} eV.

Solution for $\epsilon_{\rm m}(T)$

Adiabaticity: $\rho_{\rm de}$ and $p_{\rm de}$ can be calculated using equilibrium statistical mechanics.
Using $\rho(u)$ and $p(u)$ in the energy conservation equation and considering that $T = T(t)$ and $\epsilon_{\rm m} = \epsilon_{\rm m}(t)$, after some algebra:

$$(e^{\epsilon_{\rm m}/T} + 1)\frac{\dot{\epsilon}_{\rm m}}{\epsilon_{\rm m}} = \left(1 + \frac{T}{\epsilon_{\rm m}}\right)\frac{\dot{T}}{T}, \tag{6}$$

or, in terms of u and a,

$$\frac{(1 + e^u)}{(1 - u\, e^u)}\dot{u} = -\frac{\dot{a}}{a}.$$

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The solution is

$$\frac{\epsilon_{\rm m}}{T} = \frac{\epsilon_\infty}{T} + e^{-\epsilon_{\rm m}/T}, \tag{8}$$

where ϵ_∞ is the (constant) value of $\epsilon_{\rm m}$ in the limit of small temperature, $T \ll \epsilon_\infty$, $a \to \infty$, $t \to \infty$.

Small temperature limit $T \ll \epsilon_\infty$

In this limit we have $u \gg 1$ and $\epsilon_{\rm m} = \epsilon_\infty$. We obtain:

$$\rho_{\rm de} = \frac{g\epsilon_\infty^4}{8\pi^2}; \qquad p_{\rm de} = -\frac{g\epsilon_\infty^4}{8\pi^2}; \qquad w = -1$$

so ewkons behave as a cosmological constant.

- Assuming $k = 0$ and using H_0 and $\Omega_{\rm de} \simeq 0.68$ from the Planck Collaboration, then $\rho^0_{\rm de} = 2.53\;10^{-11}$ eV^4.

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- In a present small-temperature regime: $\epsilon_\infty = 0.0067eV$ for $g = 1$, or $0.0038eV$ for $g = 10$, i.e. between 16 to 28 times larger than T_0 (assuming $T_0 = T_0^{CMB}$, large enough to neglect the exponential term in Eq. (8)).

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- Thus $\rho_{\rm de} \simeq const = \rho^0_{\rm de} \simeq \rho^\infty_{\rm de}$, so

$$\rho_\infty \equiv \rho^\infty_{\rm de} = \frac{g\epsilon_\infty^4}{8\pi^2}. \tag{9}$$

Large temperature, $T \gg \epsilon_\infty$

We can neglect the first term in the right-hand side of Eq. (8):

$$\frac{\epsilon_{\rm m}}{T} = e^{-\epsilon_{\rm m}/T}, \tag{10}$$

Solution: $\epsilon_{\rm m} = 0.567\,T$ (so $u \to \infty$ for $t \to \infty$ and $u \to 0.567$ for early times). Then we obtain:

$$\rho_{\rm de} = 2.15\;10^{-3}\,gT^4 \tag{11}$$

$$p_{\rm de} = 0.715\;10^{-3}\,gT^4 \tag{12}$$

$$w = 1/3, \tag{13}$$

where the numbers in Eqs. (11) and (12) can be computed with arbitrary precision. Then for $T \gg \epsilon_\infty$ ewkons behave as radiation, with $\rho_{\rm de} \sim a^{-4}$.

Crossover values

Comparing Eqs. (9) and (11), the scale factor at the crossover between both regimes is

$$a_c = T_0 \left(\frac{2.15\ 10^{-3} g}{\rho_0} \right)^{1/4}, \tag{14}$$

corresponding to $0.04 > a > 0.02$ $(24 < z < 49)$ for $10 > g > 1$. The range of the crossover temperature is 0.01 eV$< T <$ 0.006 eV $(70K < T < 115K)$.

Evolution of w

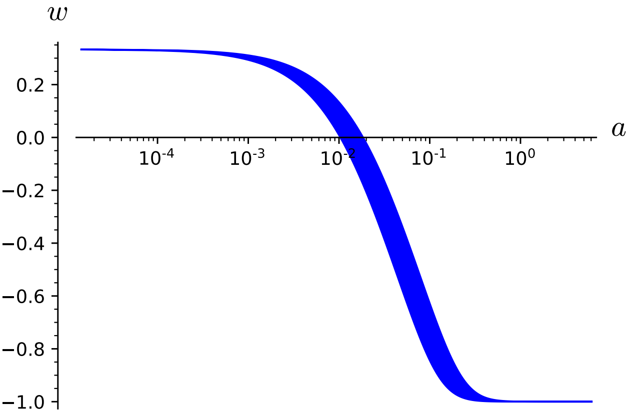

Figure: $w(a)$ for dark energy with ewkon statistics in log scale (Eq. (4)); $u = \epsilon_{\mathrm{m}}/T$ obtained from Eq. (8); $T = T_0/a$.
The thickness of the curve represents $1 < g < 10$. w takes the asymptotic values $1/3$ and -1 for small a and large a respectively.

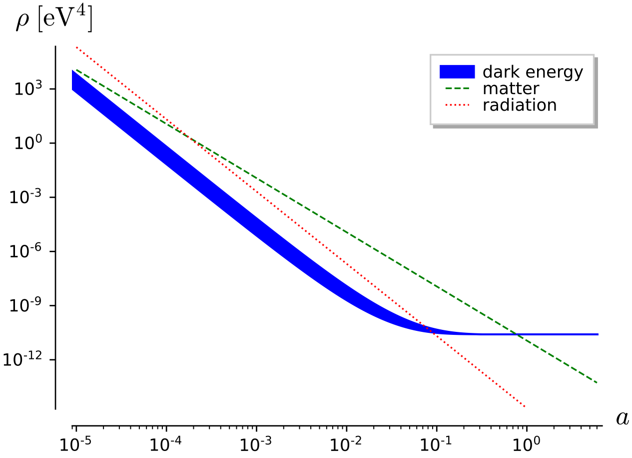

Figure: Density against scale factor in log-log scale. The density of dark energy (blue) is obtained from Eq. (2) with u calculated from Eq. (8); the thickness of the curve corresponds to $1 < g < 10$. Densities of matter (dashed line) and radiation (dotted line) are also shown for comparison.

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- Although the density of ewkons increases in the past, it never dominates in that epoch.
- These results are consistent with the hot big bang theory: a radiation-dominated universe followed by a matter-dominated universe.

The effective scalar field description of ewkons

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- The idea of representing statistical effects with an effective potential has been applied to fermions aswell as bosons.
- It is important to keep in mind that particles are non-interacting and the effective potential $V(\phi)$ is chosen just to reproduce the statistical effects of ewkons. We have

$$-V' \equiv -\frac{\partial V}{\partial \phi} = \ddot{\phi} + 3H\dot{\phi}, \tag{15}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V, \tag{16}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V. \tag{17}$$

Using Eqs. (2) and (3):

$$V = \frac{\rho_\phi - p_\phi}{2}$$
$$= \rho_\infty \frac{u^4 + 8 - 2\left(u^3 + 2\,u^2 + 4\,u + 4\right)e^{-u}}{(u - e^{-u})^4} \xrightarrow[u\to\infty]{} \rho_\infty, \qquad (18)$$

where ρ_∞ was defined in (9) and from Eq. (8) we obtained:

$$T = \frac{\epsilon_\infty}{u - e^{-u}}. \qquad (19)$$

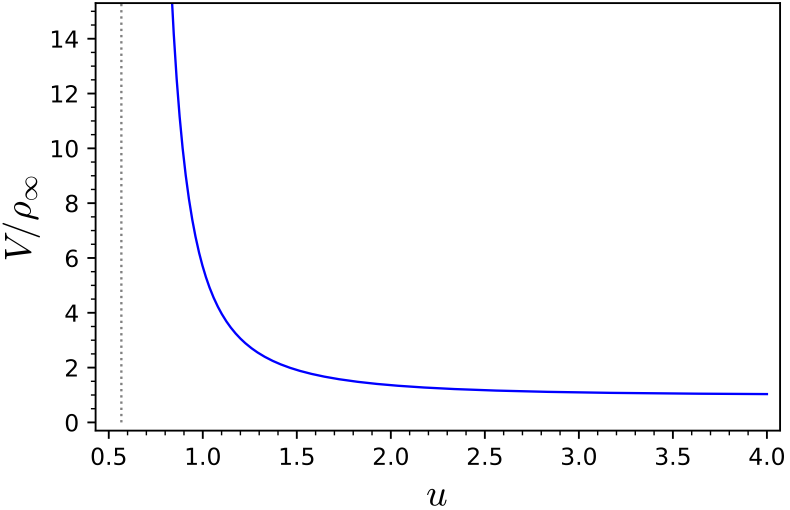


Figure: The potential V as a function of $u = \epsilon_{\rm m}/T$; it diverges at $u \simeq 0.567$ (dotted vertical line). Recall that this (constant) asymptotic value of u corresponds to $T \to \infty$ ($a \to 0$).

To obtain V' we see that:

$$V' = \frac{dV}{du}\frac{du}{da}\frac{da}{d\phi}. \tag{20}$$

$u(a)$ and a is given by (19) and $T = T_0/a$:

$$a = \alpha(u - e^{-u}), \tag{21}$$

where $\alpha \equiv T_0/\epsilon_\infty$ is a constant. Then:

$$\frac{du}{da} = \frac{1}{\alpha(1 + e^{-u}).} \tag{22}$$

Given that $\dot{\phi}^2 = \rho_\phi + p_\phi$, then (we choose the positive square root for $\dot{\phi}$; we clarify this point below)

$$: \frac{da}{d\phi} = \frac{\dot{a}}{\dot{\phi}} = \frac{aH}{\sqrt{\rho_\phi + p_\phi}}, \tag{23}$$

Thus we have a, ρ_ϕ and p_ϕ in terms of u.
From (22) and (23) we have also du/da and $da/d\phi$ in terms of u.
Using them in (20) we obtain also V' as a function of u.
This process can be repeated to obtain

$$V'' = \frac{dV'}{du}\frac{du}{da}\frac{da}{d\phi}, \tag{24}$$

etc. These (large) expressions can be calculated with a computational algebra software, including the limit of the derivatives of the potential when $u \to \infty$.

Dark energy dominated era

The Friedmann equation is $H^2 = \rho_{\rm tot}/(3m_P^2) \simeq \rho_{\rm de}/(3m_P^2)$
$m_P = 1/\sqrt{8\pi G}$ is the reduced Planck mass and the total density $\rho_{\rm tot}$ is similar to $\rho_{\rm de}$ in the dark energy dominate era.
From (23)

$$\frac{da}{d\phi} = \frac{\alpha(u - e^{-u})}{\sqrt{3}\, m_P \sqrt{1+w}}, \tag{25}$$

where Eq. (21) was used for a, and $w(u)$ was written in (4).

Sequence obtained for the derivatives of the potential:

$$\begin{aligned} V_\infty^{(n)} &= 0 & \text{for } n \text{ odd}, \\ V_\infty^{(n)} &= \tfrac{\rho_\infty}{6}\left(\tfrac{2}{m_P}\right)^n & \text{for } n \text{ even}, \end{aligned} \quad (26)$$

where "∞" $u \to \infty$.
The sequence was numerically checked up to $n = 20$.
For simplicity we take $\phi_\infty = 0$, when $u \to \infty$.
Using the Taylor expansion

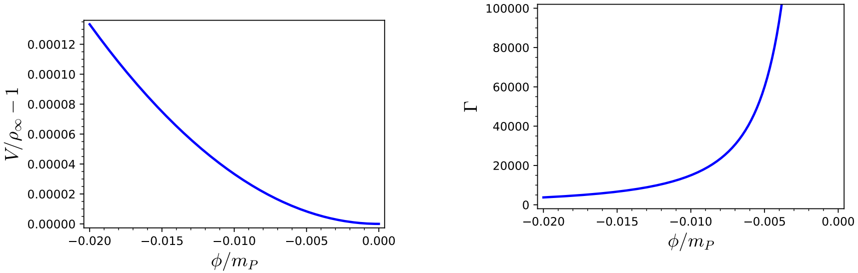
$$V = V_\infty + V'_\infty \phi + V''_\infty \phi^2/2! + \cdots , \quad (27)$$

we obtain

$$V(\phi) = \frac{\rho_\infty}{6}\left[5 + \cosh\left(\frac{2\phi}{m_P}\right)\right] . \quad (28)$$

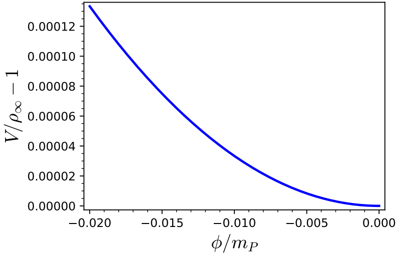
, where the ewkons approach a cosmological constant like behavior in the long run. (Scherrer-2008, Chiba-2009, Gong-2014, Gupta-2015, You-2023)

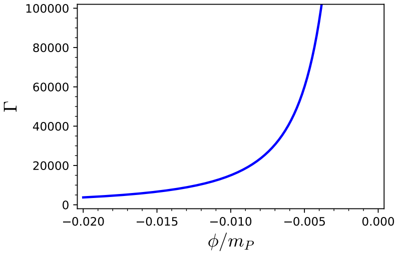
$V(\phi)$ and $\Gamma(\phi)$ in the dark energy dominated era



- Γ diverges when $\phi \to \phi_\infty = 0$ at the bottom of the potential.

$V(\phi)$ and $\Gamma(\phi)$ in the dark energy dominated era





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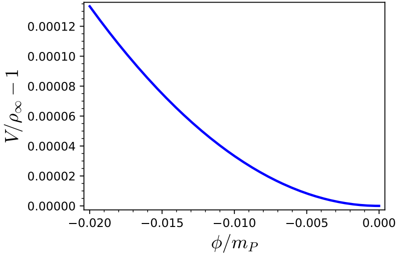

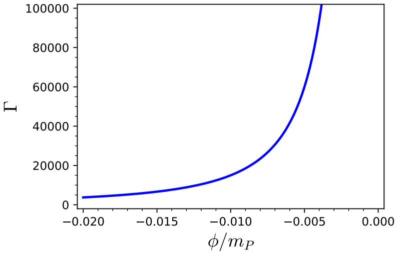

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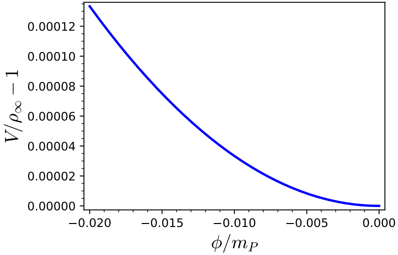

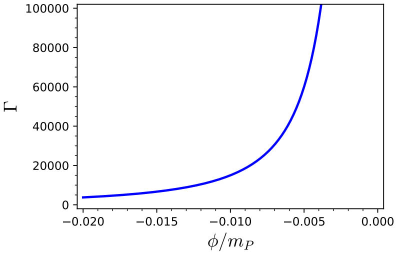

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- Initial conditions $\phi > \phi_\infty = 0$ and $\phi < \phi_\infty = 0$ are equivalent.

Dynamics of the scalar field ϕ

- We solve the Klein-Gordon equation (15) with $V(\phi)$ for $\phi(t)$, to verify that $V(\phi)$ correctly reproduces the behavior we obtained for small T, a condition that approximately overlap with the regime of dominant ewkons.

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- If ewkons already account for the bulk of dark energy, the cosmology studied would be valid from the present time throughout the far future of the Universe.
- Recall that Ewkons are described by a gas of ultra relativistic particles at temperature T.

Scalar field description of dark energy domination (small temperature):

We have $\epsilon_{\rm m} \simeq \epsilon_\infty$, $u \simeq \epsilon_\infty/T \gg 1$, $a \gg 1$ and $|\phi|/m_P \ll 1$ $(\Omega_{\rm de} = 1)$.
Eqs. (18) and (28) give $V(u)$ and $V(\phi)$ respectively.
Approximating both equations in the present regime:

$$V = \rho_\infty(1 + 8/u^4) \tag{29}$$
$$V = \rho_\infty \left[1 + \phi^2/(3m_P^2)\right] \tag{30}$$

and, combining them,

$$\frac{\phi}{m_P} = \frac{\sqrt{24}}{u^2} = \frac{\sqrt{24}\, T^2}{\epsilon_m^2} \simeq \frac{\sqrt{24}\, T_0^2}{\epsilon_\infty^2}\frac{1}{a^2}, \tag{31}$$

where both signs of the square root are possible.

No slow roll

From this equation, we obtain:

$$\dot{\phi} = -2\phi H \quad (32)$$

$$\ddot{\phi} = -2\dot{\phi} H \quad (33)$$

where in the last equation the term $-2\phi\dot{H}$ was neglected, since it can be shown that $-2\phi\dot{H} \propto 1/a^6$.

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- Therefore $\ddot{\phi}$ cannot be neglected in (15) so, although the kinetic energy of the scalar field may be much less than V, the slow roll approximation does not fully hold.

The evolution equation now becomes (recall that we have not neglected the $\ddot{\phi}$ term, but absorbed it in the $\dot{\phi}$ one):

$$H\dot{\phi} + V' = 0. \tag{34}$$

Keeping terms up to order $1/a^2$, $H \simeq \sqrt{\rho_\infty}/(\sqrt{3}m_P)$, where, since $\Omega_{\rm de} = 1$, we have taken $\rho_{\rm tot} = \rho_{\rm de} \simeq \rho_\infty$. Then,

$$\dot{\phi} + \lambda\phi = 0, \tag{35}$$

with

$$\lambda \equiv \frac{2}{m_P}\sqrt{\frac{\rho_\infty}{3}} = \sqrt{\frac{32}{3}\pi G \rho_\infty}. \tag{36}$$

Exponential decay of the scalar field

$$\phi = \phi_0 \exp\left[-\lambda(t - t_0)\right] \tag{37}$$

with $\phi_0 \equiv \phi(t_0)$.
We confirm that the asymptotic value of the scalar field coincides with the minimum of the potential $\phi_\infty = 0$.
Replacing $a = 1$ in (31):

$$\frac{\phi_0}{m_P} = \frac{\sqrt{24}\, T_0^2}{\epsilon_\infty^2}. \tag{38}$$

Using the values of ϵ_∞ computed previously, $|\phi_0|/m_P \simeq 0.006$ for $g = 1$, and $\simeq 0.02$, for $g = 10$.

Exponential increase of the scale factor

From Eq. (31):

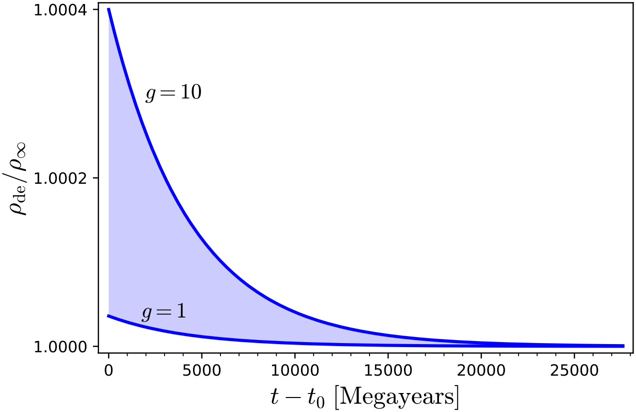
$$a = e^{\lambda(t-t_0)/2}. \tag{39}$$

From the expression for $\rho(\phi)$ ((16)):

$$\rho_{\mathrm{de}} = \rho_\infty \left(1 + \left(\frac{\phi_0}{m_P}\right)^2 e^{-2\lambda(t-t_0)}\right), \tag{40}$$

confirming that, at present, ρ_0 is approximately equal to ρ_∞.

Future evolution of $\rho_{\rm de}/\rho_\infty$ (Eq. (40)).



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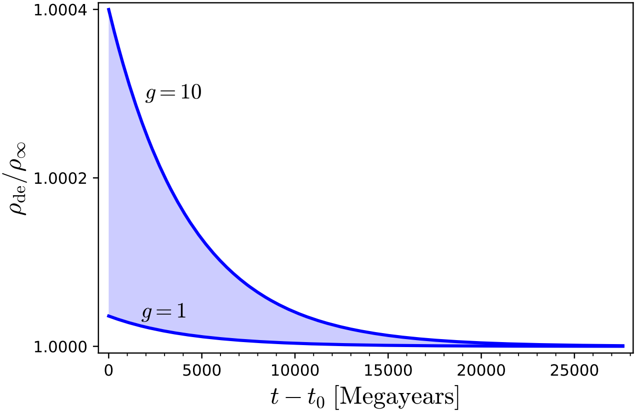

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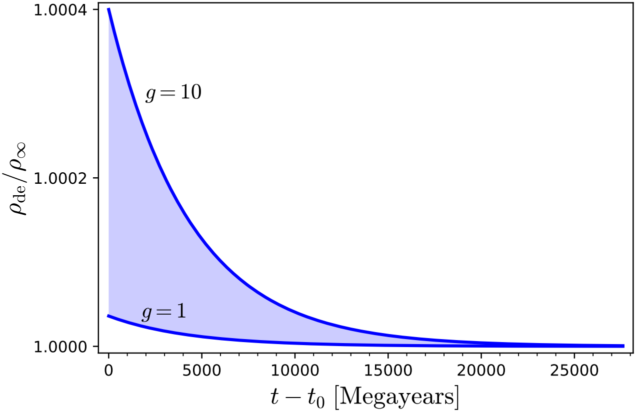

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- The decay constant $\lambda \simeq (8740 \text{ Myr})^{-1}$ (from Eq. (36)).

Conclusions

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- Under the assumption of energy conservation, the cut-off energy is a time-dependent quantity.
- These particles have a dark-energy type equation of state, which makes them a possible explanation of the accelerated expansion of the Universe.
- In the case of massless ewkons, the solution has the remarkable property that the presence of ewkons remains almost unnoticed until recent times, when the Universe becomes ewkon-dominated.

- To compare our proposal with current literature, we derived a scalar field effective picture of the scenario, and compared the potential obtained with other models.

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- The potential corresponds to an effective description of these quasi-indistinguishable particles that represents statistical effects; however, we should keep in mind that these particles are non-interacting.
- This is a substantially different proposal from the current literature, being based as it is on non-trivial statistical assumptions.

Prospects and issues to be clarified

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- Given the great generality of this approach, early dark energy models (see e.g. Kojima-2022) can be also included in our analysis, by choosing conveniently the ewkon parameters.
- However, we think it desirable to better understand the theoretical basis of ewkons, before looking for any scalar field effective description during the matter and radiation-dominated eras.

- This is further required, considering the lack of evidence for extensions to the ΛCDM model (see e.g. Heavens-2017) and the delicate observational issues involved in the current cosmological tensions (see e.g. Abdalla-2022).

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- Finally, if we consider ϵ_∞ as the only parameter to be adjusted in our model, then we can see our approach as belonging to the *one-parameter dynamical dark-energy parametrizations*.
- However, while these models usually involve the present barotropic parameter w_0 as the only free parameter to be adjusted observationally with different *ad-hoc* functions $w(a)$ (see e.g. Yang-2019) for a list of five such functions), ours predict a definite cosmological evolution for it, making it particularly interesting from a dynamical/theoretical point of view.

Thank you