15 Mayo 2024

Correladores fuera de tiempo: caos, integrabilidad, signos clásicos de aproximación al equilibrio

Ignacio García-Mata
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ARGENTINA
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OTOC & dinamica subyacente

sistemas simples

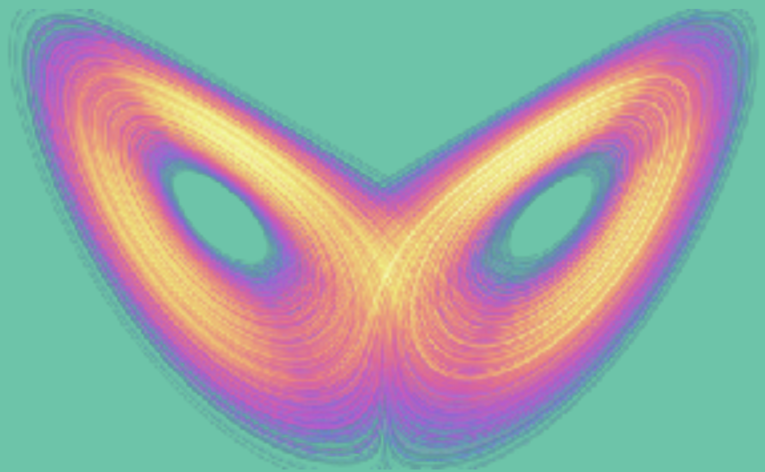
con y sin análogo clásico

“If there exist fewer than d [constants of motion], as is the case, for example, according to Poincaré in the three-body problem, then the p_i are not expressible by the q_i and the quantum condition of Sommerfeld–Epstein fails also in the slightly generalized form that has been given here.”

— Einstein, DPG meeting 1917

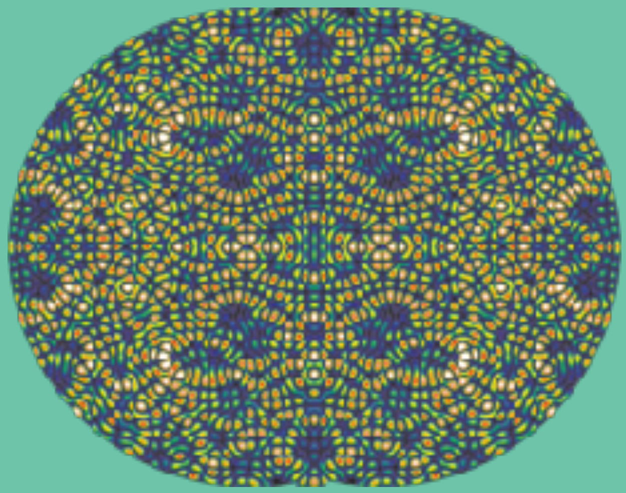
“Einstein’s Unknown Insight and the Problem of Quantizing Chaos” D. Stone, Physics Today (2005)

caos clásico



“Caos cuántico”

sistemas clásicamente caóticos, cuantizados



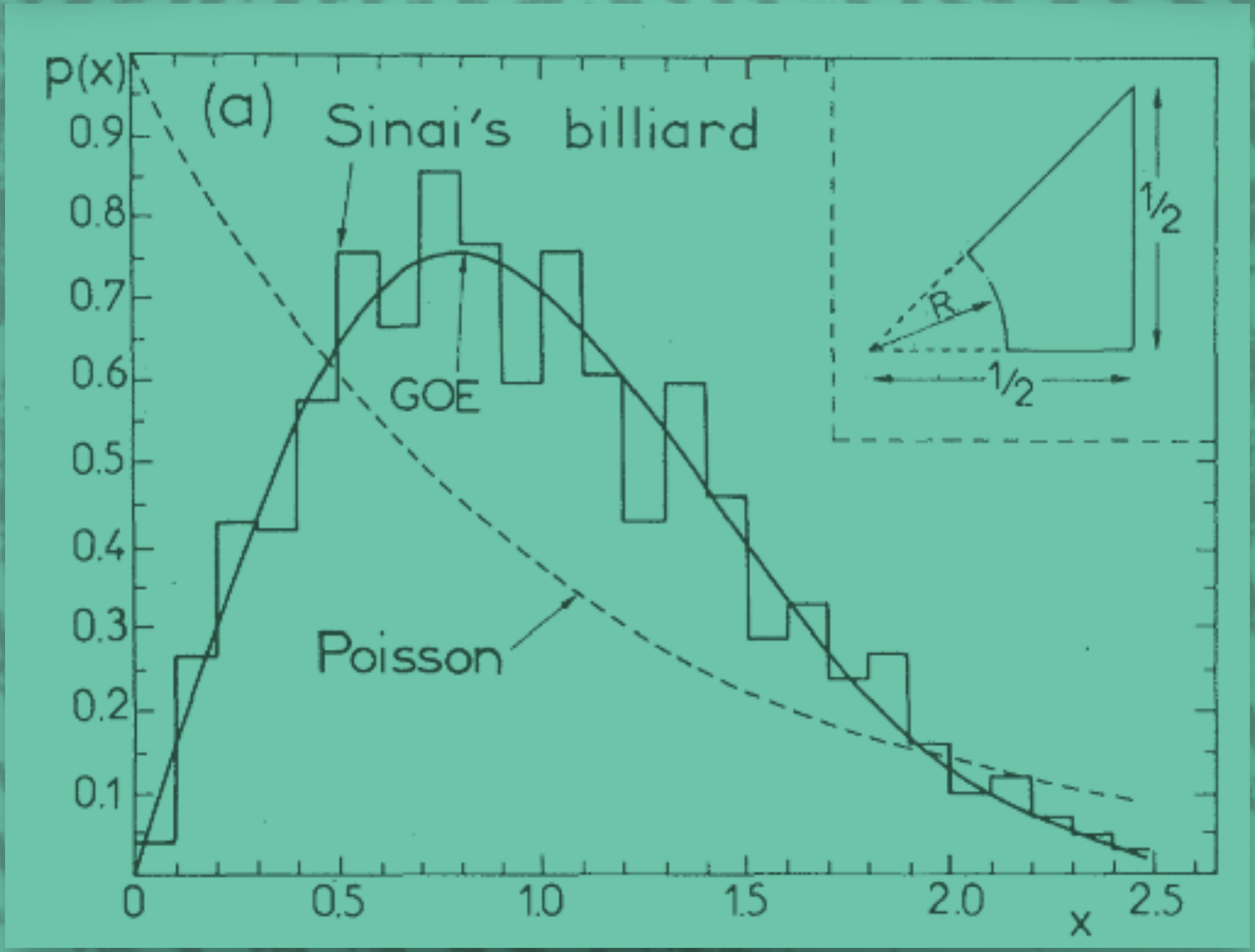
identificar
el caos cuántico

Estadística espectral
matrices aleatorias

O. Bohigas, M. J. Giannoni, and C. Schmit

PHYSICAL REVIEW LETTERS

Volume 52 | 2 January 1984 | Number 1



Dinámica

Peres 1984

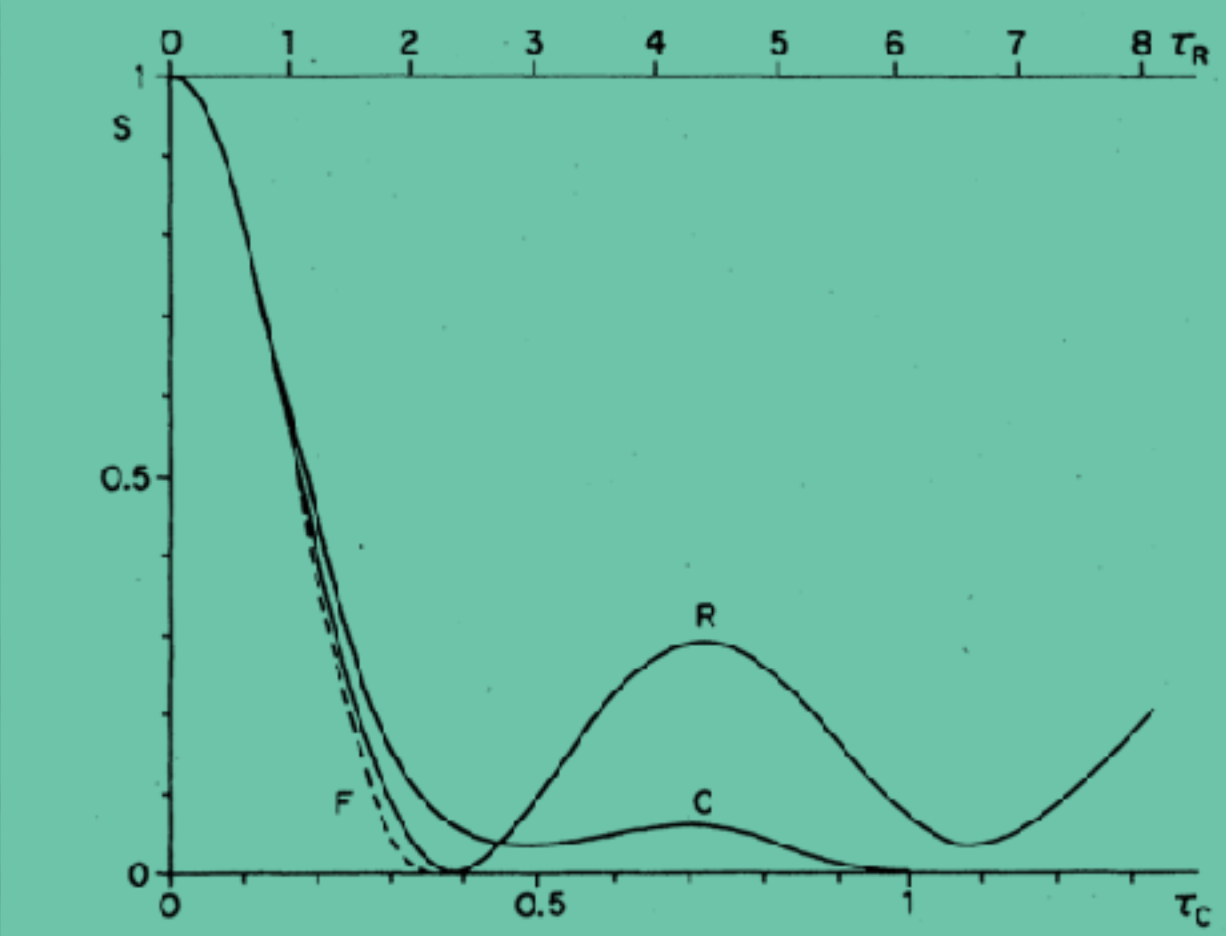
Sensibilidad a perturbaciones en el Hamiltoniano

PHYSICAL REVIEW A VOLUME 30, NUMBER 4 OCTOBER 1984

Stability of quantum motion in chaotic and regular systems

Asher Peres
Department of Physics, Technion—Israel Institute of Technology, Haifa 32000, Israel

(Received 4 June 1984)



Dinámica

Peres 1984

Sensibilidad a perturbaciones en el Hamiltoniano

🇺🇾 🇦🇷

Jalabert & Pastawsky 2000

“Loschmidt echo”

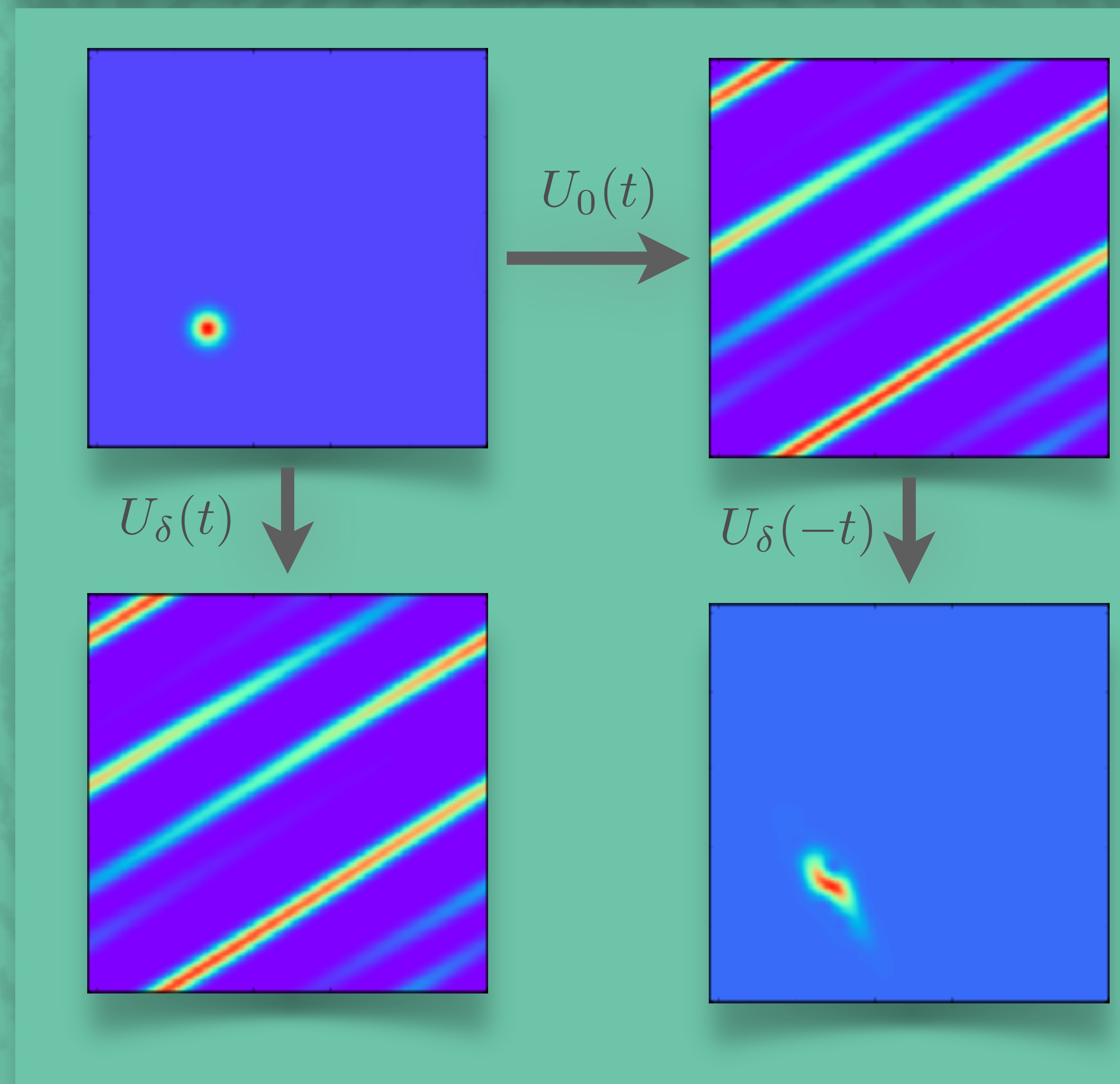
$$M(t) = |\langle\psi|U_\delta(-t)U_0(t)|\psi\rangle|^2$$

régimen (Lyapunov) independiente de la perturbación


VOLUME 86, NUMBER 12 PHYSICAL REVIEW LETTERS 19 MARCH 2001

Environment-Independent Decoherence Rate in Classically Chaotic Systems

Rodolfo A. Jalabert[1] and Horacio M. Pastawski[1,2]



Huella digital del Caos Cuántico

Espectro	Estadistica de niveles gap ratios
dynamics	Loschmidt echo Survival probability
estructura de autoestados	localización termalización entropía de entanglement

16’ **OTOCS:** out of time ordered correlators

Larkin and Ovchinnikov 69

Maldacena, Shenker & Stanford (JHEP 2016)

$$\mathcal{C}(t) = \left\langle\left\langle \left[\hat{A}_t, \hat{B}\right]^\dagger \left[\hat{A}_t, \hat{B}\right] \right\rangle\right\rangle = \left\langle\left\langle \left|\left[\hat{A}_t, \hat{B}\right]\right|^2 \right\rangle\right\rangle$$

$$\left\langle\left\langle \hat{O} \right\rangle\right\rangle = \frac{1}{Z}\,\mathrm{Tr}\left\{e^{-\beta\hat{H}}\hat{O}\right\}$$

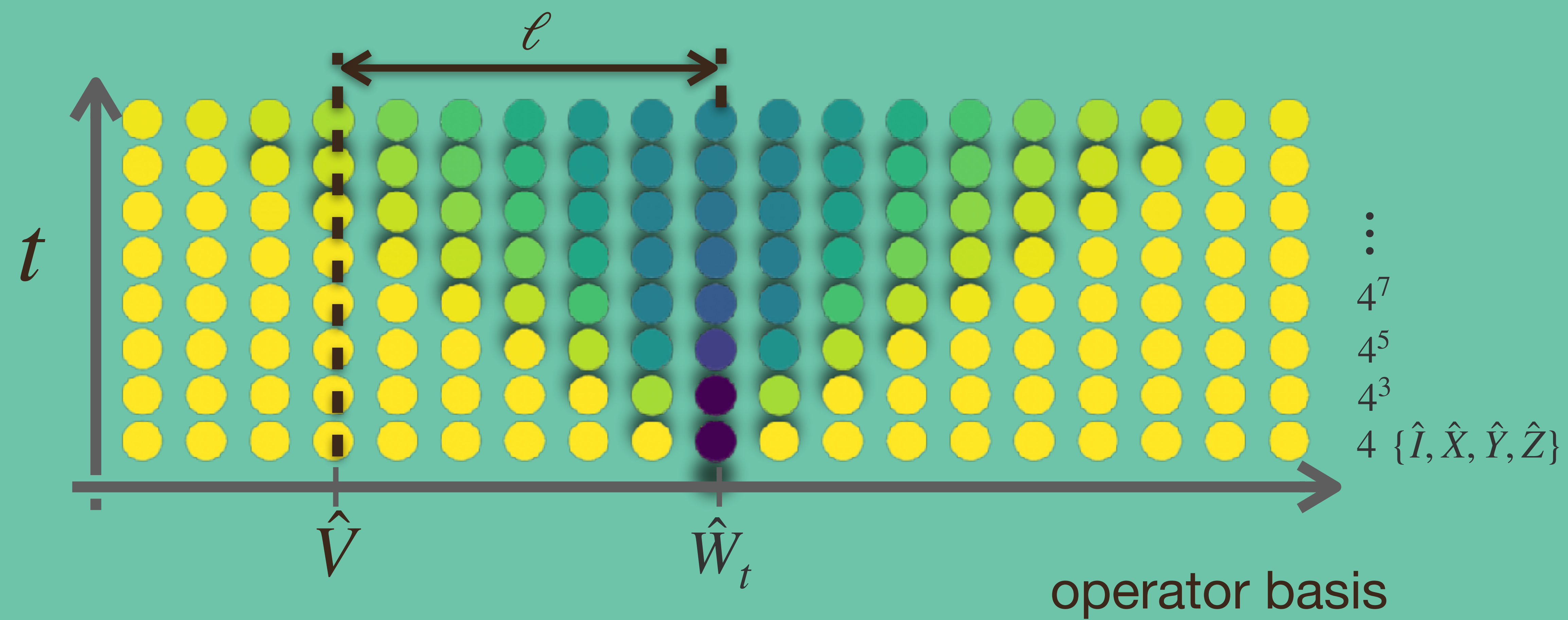
$\hat{A},\ \hat{B}$ Hermitian, then $C(t) = 2(1 - \mathrm{Re}[\langle\langle \hat{A}_t\, \hat{B} \hat{A}_t\, \hat{B} \rangle\rangle]$

algunos llaman a esto OTOC

yo lo llamo $O_1(t)$

efectos de una perturbación B en la evolución de A: operator growth

Scrambling describe el proceso por el cual una perturbación inicialmente localizada se desparrama sobre los grados de libertad en un sistema complejo



$$W(t) = \underbrace{e^{iHt}}_{\text{backward}} \underbrace{W}_{\text{perturbation}} \underbrace{e^{-iHt}}_{\text{forward}}$$

$$W(t) = \sum_{\ell=0}^{\infty} \frac{(it)^{\ell}}{\ell\,!}[H, \ldots [H, W], \ldots]$$

Fast Scramblers

(Sekino and Susskind, 2008)

$$C(t) \sim e^{\lambda t}$$

Black holes

bound on chaos

Maldacena, Shenker & Stanford (JHEP 2016)

$$\lambda \leq 2\pi \frac{k_B T}{\hbar}$$

ADS/CFT correspondence

Sachdev-Ye-Kitaev saturates bound

Quantum information scrambling

thermalization and fluctuation theorems

entanglement witness

many-body localization

ESQPT

Lyapunov

$$\left[\hat{X}_t, \hat{P}\right] \to \text{Poisson bracket} \;= i\hbar \frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda t}$$

$$\mathcal{C}(t) = -\left\langle \left[\hat{X}_t, \hat{P}\right]^2 \right\rangle \sim e^{2\lambda t}$$

Experimentos

Gärttner, M. et al. Measuring out-of-time-order correlations and multiple quantum spectra in a trapped-ion quantum magnet. Nat. Phys. 13, 781–786 (2017).

Li, J. et al. Measuring out-of-time-order correlators on a nuclear magnetic resonance quantum simulator. Phys. Rev. X 7, 031011 (2017).

Wei, K. X., Ramanathan, C. & Cappellaro, P. Exploring localization in nuclear spin chains. Phys. Rev. Lett. 120, 070501 (2018).

Meier, E. J., Ang'ong'a, J., An, F. A. & Gadway, B. Exploring quantum signatures of chaos on a Floquet synthetic lattice. Phys. Rev. A 100, 013623 (2019).

Mi, X., Roushan, P., Quintana, C., Mandra, S., Marshall, J., Neill, C., ... & Chen, Y. (2021). Information scrambling in quantum circuits. Science, 374(6574), 1479-1483.

M. S. Blok, V. V. Ramasesh, T. Schuster, K. O'Brien, J. M. Kreikebaum, D. Dahlen, A. Morvan, B. Yoshida, N. Y. Yao, and I. Siddiqi. Quantum Information Scrambling on a Superconducting Qutrit Processor. Phys. Rev. X 11, 021010 (2021)

⋮

seguro hay muchos más

tiempos chicos

caso más simple

Mapas en el toro

clasico

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = F\left[\begin{pmatrix} q \\ p \end{pmatrix}\right] \quad \mathrm{mod}\ 1$$

lineal: mapa del gato

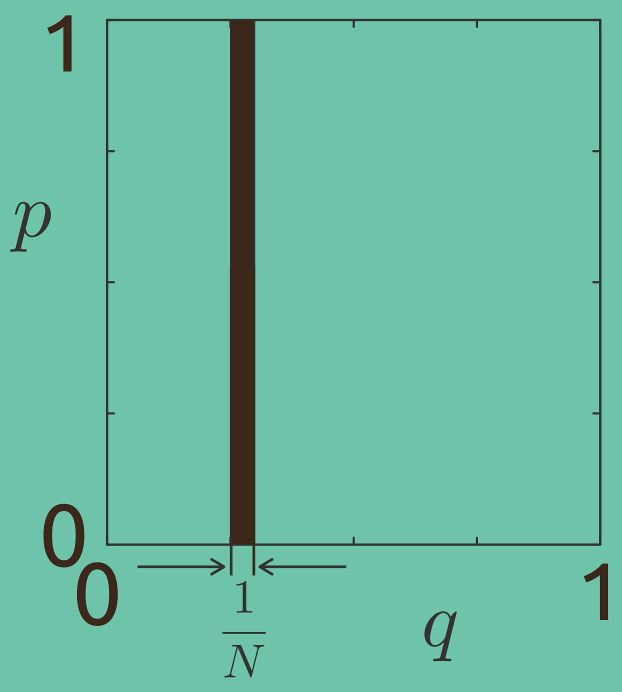
lineal por partes: panadero

no lineal: standard, Harper…

cuántico

unitary $U\ (N \times N)$

$$\hbar_{\mathrm{eff}} = 1/(2\pi N)$$



$\tilde{\phi}$, and then $\tilde{\phi}^2$ as pictured in Figure (1.17). The linear mapping $\tilde{\phi}$ has two real proper values λ_1 and λ_2: $0 < \lambda_2 < 1 < \lambda_1$.

Gato de Arnold

V. I. Arnold and A. Avez, Ergodic Problems of Classical Mechanics (1968)

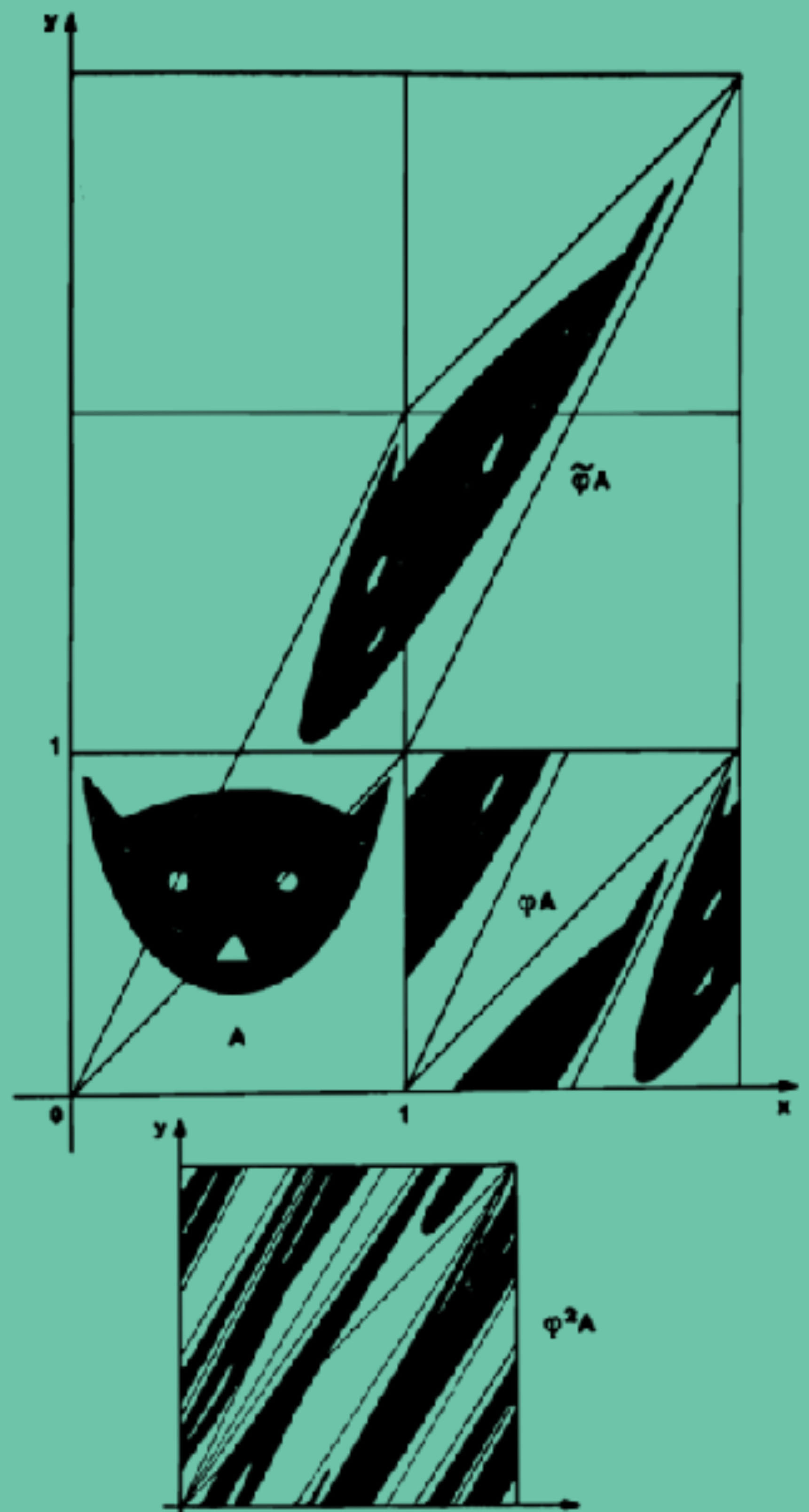


Figure 1.17

mapa del gato

$$\begin{pmatrix} q' \\ p' \end{pmatrix} = M \begin{pmatrix} q \\ p \end{pmatrix} \qquad M = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

Lyapunov $\lambda = \ln\left(\frac{3+\sqrt{5}}{2}\right)$

gato perturbado

$$\begin{aligned} p' &= p + q - 2\pi k \sin[2\pi q] \\ q' &= q + p' + 2\pi k \sin[2\pi p'] \end{aligned} \quad \mathrm{mod}\ 1.$$

gato perturbado cuántico

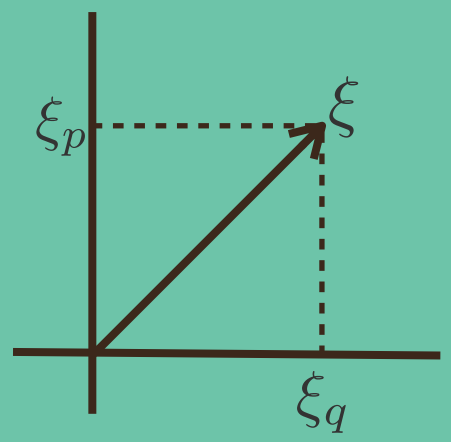
$$\hat{U}_{pcat} = e^{-i2\pi(\frac{p^2}{2N} - kN\cos(2\pi p/N))} e^{-i2\pi(\frac{q^2}{2N} + kN\cos(2\pi q/N))}$$

$F^\dagger U_2 F U_1 |\psi\rangle$ mapas pateado: standard, Harper …

eficiente numéricamente

Traslaciónes

$$\hat{T}_\xi = \hat{V}^{\xi_q}\hat{U}^{\xi_p}\tau^{\xi_q\xi_p} \qquad \tau = e^{i\pi/N}$$



$$\hat{T}_\xi\hat{T}_\chi = \tau^{<\xi,\chi>}\hat{T}_{\xi+\chi}$$

$$\left[\hat{T}_\xi, \hat{T}_\chi\right] = 2i\sin\left(\frac{\pi}{N} < \xi, \chi >\right)\hat{T}_{\xi+\chi},$$

$$\hat{T}_{M^t\xi} = \hat{U}_M^{\dagger t}\hat{T}_\xi\hat{U}_M^t.$$

transforman clásicamente

definimos $\hat{F}_\xi = \dfrac{\hat{T}_\xi - \hat{T}_\xi^\dagger}{2i}$, hermíticos

$$\left[\hat{U}^t \frac{\hat{T}_\xi - \hat{T}_\xi^\dagger}{2\mathrm{i}} \hat{U}^\dagger, \frac{\hat{T}_\chi - \hat{T}_\chi^\dagger}{2\mathrm{i}}\right] = -\frac{1}{4}\left[\hat{T}_{M^t\xi} - \hat{T}^\dagger_{M^t\xi}, \hat{T}_\chi - \hat{T}^\dagger_\chi\right]$$

$$= \frac{\mathrm{i}}{2} \sin\left(\frac{\pi}{d} < M^t\boldsymbol{\xi}, \chi >\right)\left(\hat{T}_{M'\xi+\chi} + \hat{T}_{M'\xi-\chi} + \hat{T}^\dagger_{M'\xi+\chi} + \hat{T}^\dagger_{M'\xi-\chi}\right)$$

$$C(t) = -\frac{1}{N}\mathrm{Tr}\left([\hat{F}_\xi(t), \hat{F}_\chi]^2\right) = \sin^2(\frac{\pi}{N} < M^t\xi, \chi >)$$

$$< X, Y >= (x_1, x_2)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$$\mathcal{C}(t) = -\frac{1}{N}\mathrm{Tr}\left(\left[\hat{X}_t, \hat{P}\right]^2\right)$$

Shift operators

$$\hat{T}_{(1,0)} \equiv \hat{U} = \sum_{q\in\mathbb{Z}_N} |q\rangle\langle q| e^{2\pi q/N}$$

$$\hat{X} = \frac{\hat{U} - \hat{U}^\dagger}{2i}$$

$$\hat{T}_{(0,1)} \equiv \hat{V} = \sum_{q\in\mathbb{Z}_N} |q+1\rangle\langle q|$$

$$\hat{P} = \frac{\hat{V} - \hat{V}^\dagger}{2i}$$

$$C(t) = -\frac{1}{N}\mathrm{Tr}\left([\hat{F}_\xi(t), \hat{F}_\chi]^2\right) = \sin^2(\frac{\pi}{N} < M^t\xi, \chi >)$$

$$\hat{X} = \hat{F}_{(1,0)} \qquad \hat{P} = \hat{F}_{(0,1)}$$

$$M^t = \begin{pmatrix} a_t & b_t \\ c_t & d_t \end{pmatrix} \qquad < M^t\xi, \chi >= -a_t$$

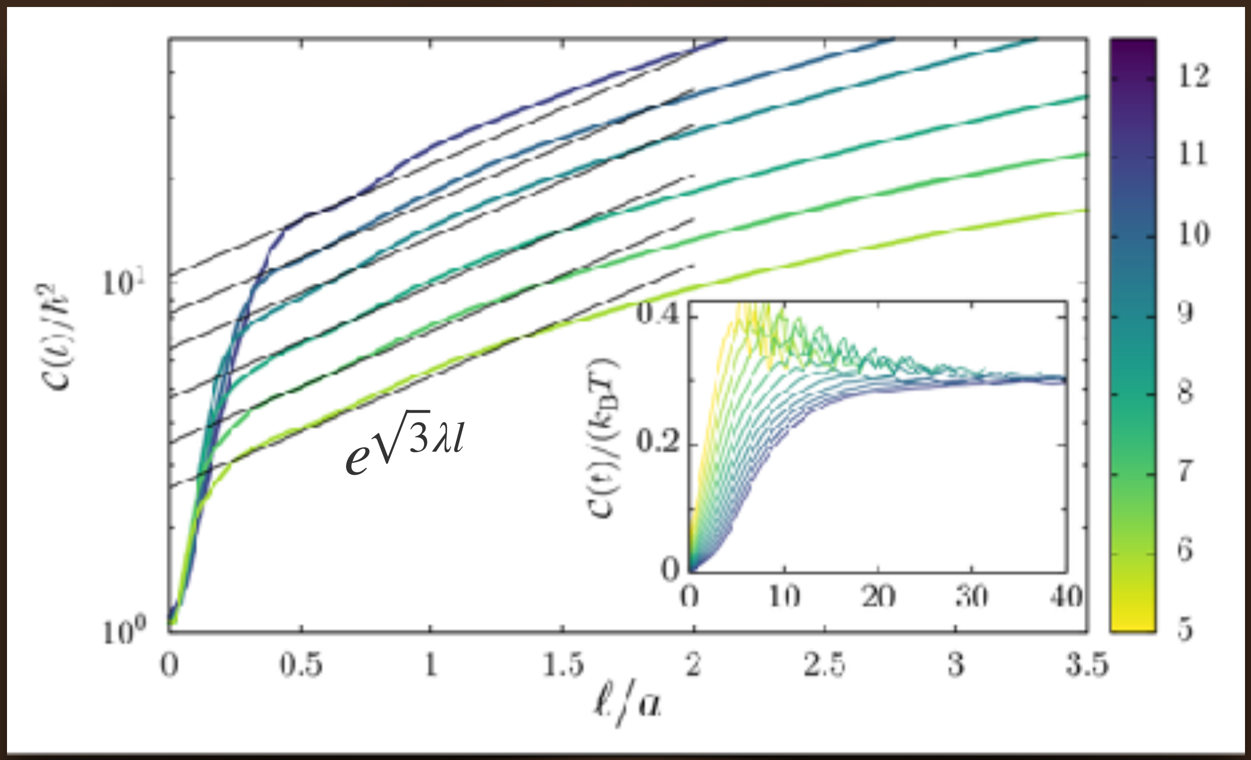
Lyapunov

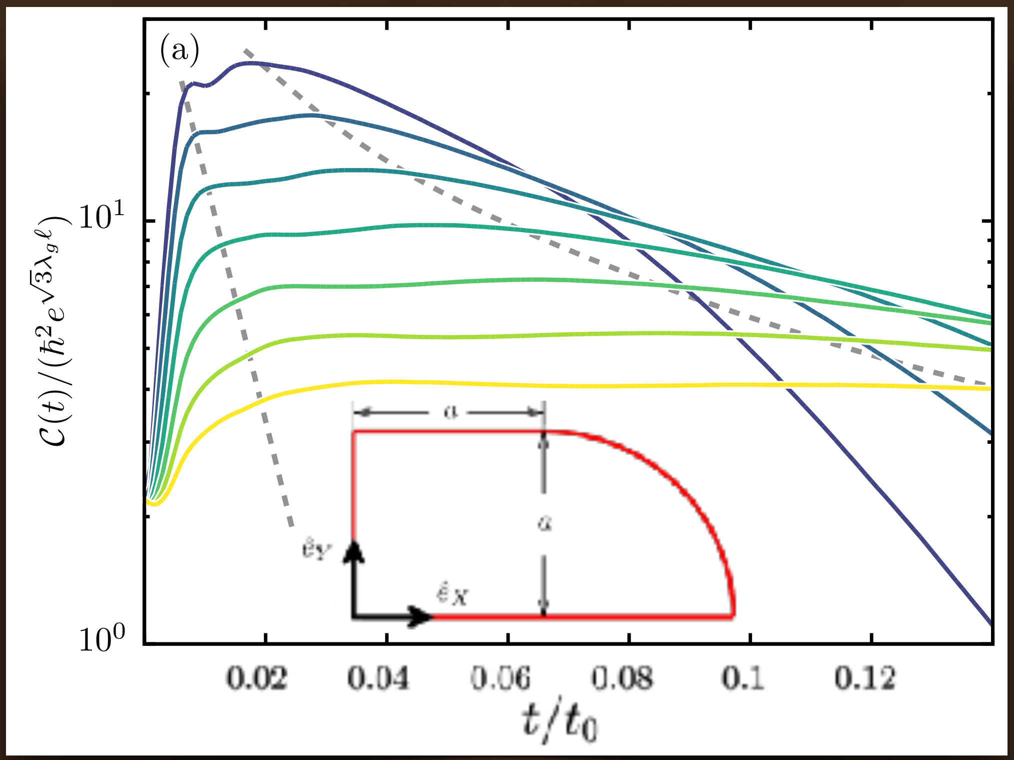
$$C(t) = \sin^2\left(\frac{\pi a_t}{N}\right) \approx \left(\frac{\pi a_t}{N}\right)^2 = \frac{\pi^2}{N^2}e^{2\lambda t}$$

analítico para el gato sin perturbar

$$< X, Y >= (x_1, x_2)\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

PHYSICAL REVIEW LETTERS **121,** 210601 (2018) enfoque semiclásico en el billar de Bunimovich





R. A. Jalabert, IGM, and D. A. Wisniacki, Phys. Rev. E 98, 062218 (2018).

one body

E. B. Rozenbaum, S. Ganeshan, and V. Galitski, Lyapunov exponent and out-of-time-ordered correlator's growth rate in a chaotic system, Phys. Rev. Lett. 118, 086801 (2017)

standard map

A. Lakshminarayan, Out-of-time-ordered correlator in the quantum bakers map and truncated unitary matrices, Phys. Rev. E 99, 012201 (2019)

baker's

J. Chávez-Carlos, B. López-del Carpio, M. A. Bastarrachea-Magnani, P. Stránsky, S. Lerma-Hernández, L. F. Santos, and J. G. Hirsch, Quantum and classical Lyapunov exponents in atom-field interaction systems, Phys. Rev. Lett. 122, 024101

Dicke

S. Sinha, S. Ray, and S. Sinha, Fingerprint of chaos and quantum scars in kicked Dicke model: an out-of-time-order correlator study, J. Phys. : Condensed Matter 33, 174005 (2021)

Dicke

Alessio Lerose and Silvia Pappalardi, "Bridging entanglement dynamics and chaos in semiclassical systems" Phys. Rev. A 102, 032404 (2020)

kicked top

many body

S. Pappalardi, A. Russomanno, B. Žunkovič, F. Iemini, A. Silva, and R. Fazio, Scrambling and entanglement spreading in long-range spin chains Phys. Rev. B 98, 134303 (2018)

N. Kolganov and D. A. Trunin, Classical and quantum butterfly effect in nonlinear vector mechanics, Phys. Rev. D 106, 025003 (2022)

coupled oscillators

B. Craps, M. De Clerck, D. Janssens, V. Luyten, and C. Rabideau, Lyapunov growth in quantum spin chains, Phys. Rev. B 101, 174313 (2020)

large spins

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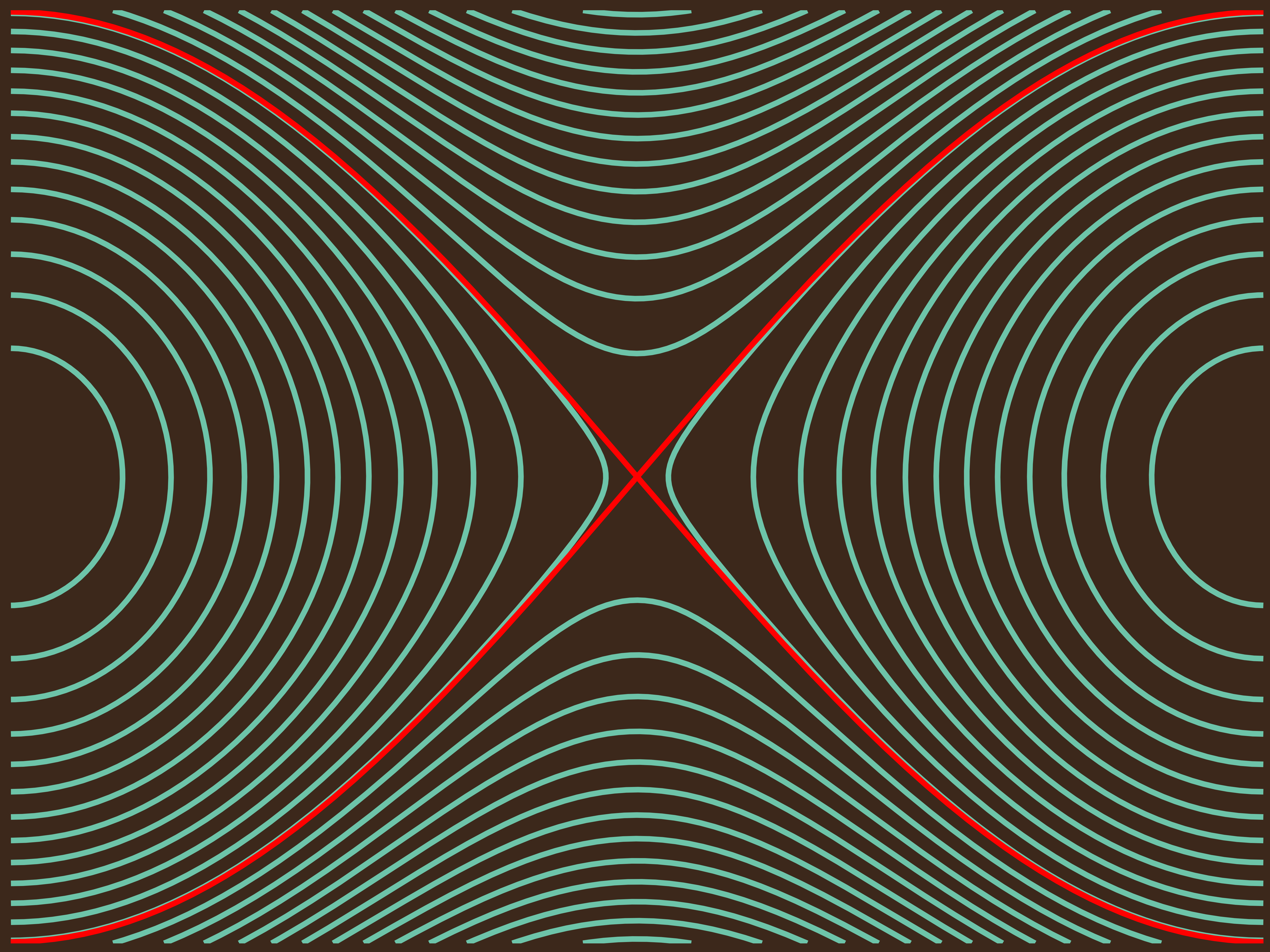
large spins

$$e^{2\lambda t} \overset{?}{\Rightarrow} \text{caos}$$

a veces


saddles



Exponential growth of out-of-time-order correlator without chaos: inverted harmonic oscillator

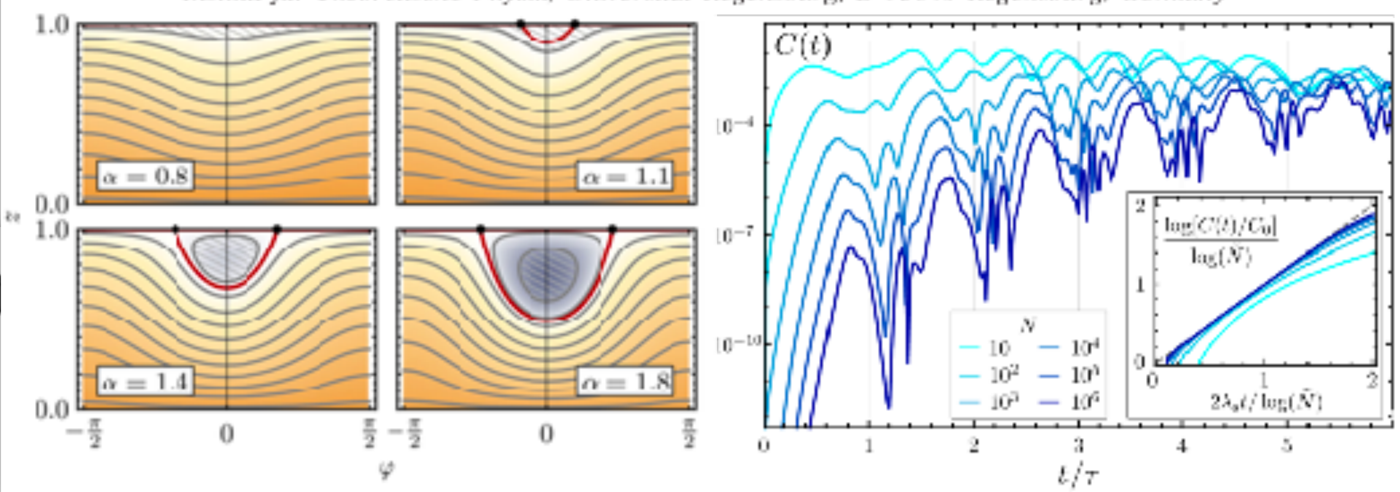
Journal of High Energy Physics, *2020*(11), 1-25.

Koji Hashimoto,[a] Kyoung-Bum Huh,[b] Keun-Young Kim,[b] Ryota Watanabe[a]

PHYSICAL REVIEW LETTERS **123**, 160401 (2019)

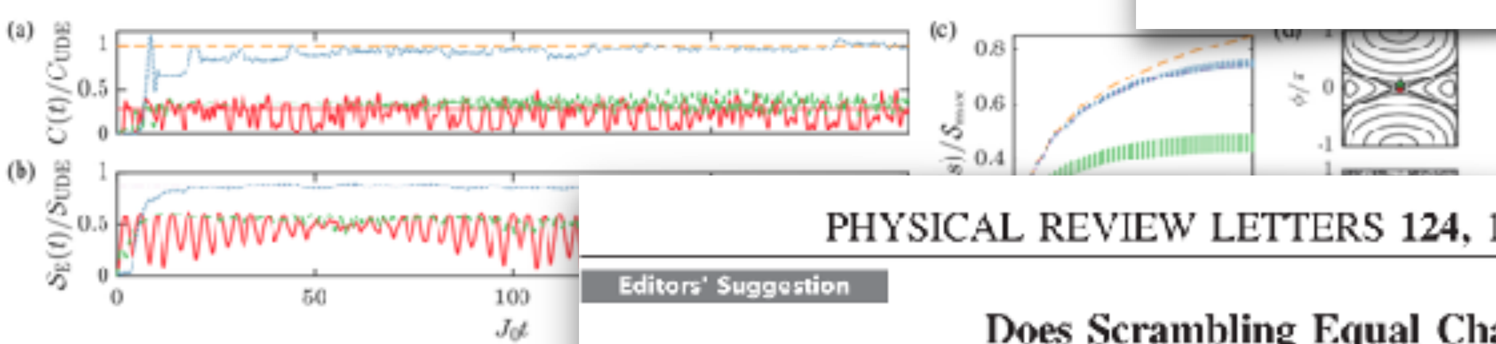
Reversible Quantum Information Spreading in Many-Body Systems near Criticality

Quirin Hummel, Benjamin Geiger,* Juan Diego Urbina, and Klaus Richter
Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany



Saddle-point scrambling without thermalizat

R. A. Kidd, A. Safavi-Naini, and J. F. Corney
Phys. Rev. A **103**, 033304 – Published 5 March 2021

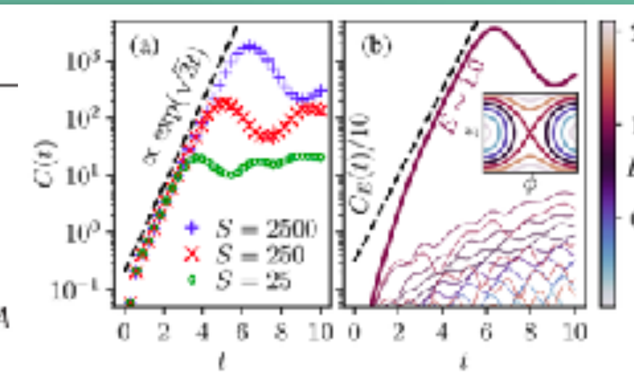


PHYSICAL REVIEW LETTERS **124**, 140602 (2020)

Editors' Suggestion

Does Scrambling Equal Chaos?

Tianrui Xu,[1,2] Thomas Scaffidi,[3] and Xiangyu Cao[1]
[1]*Department of Physics, University of California, Berkeley, California 94720, USA*
[2]*Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
[3]*Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada*

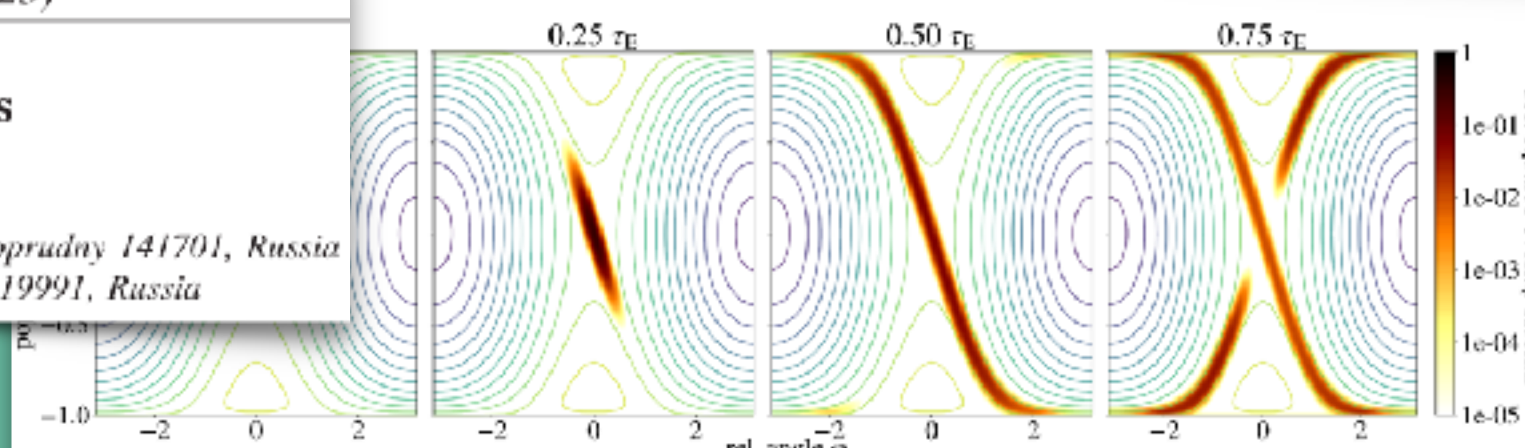


PHYSICAL REVIEW D **108**, L101703 (2023)

Letter

Quantum chaos without false positives

Dmitrii A. Trunin*
Moscow Institute of Physics and Technology, Institutskiy pereulok 9, Dolgoprudny 141701, Russia and Lebedev Physical Institute, Leninskiy prospect 53, Moscow 119991, Russia

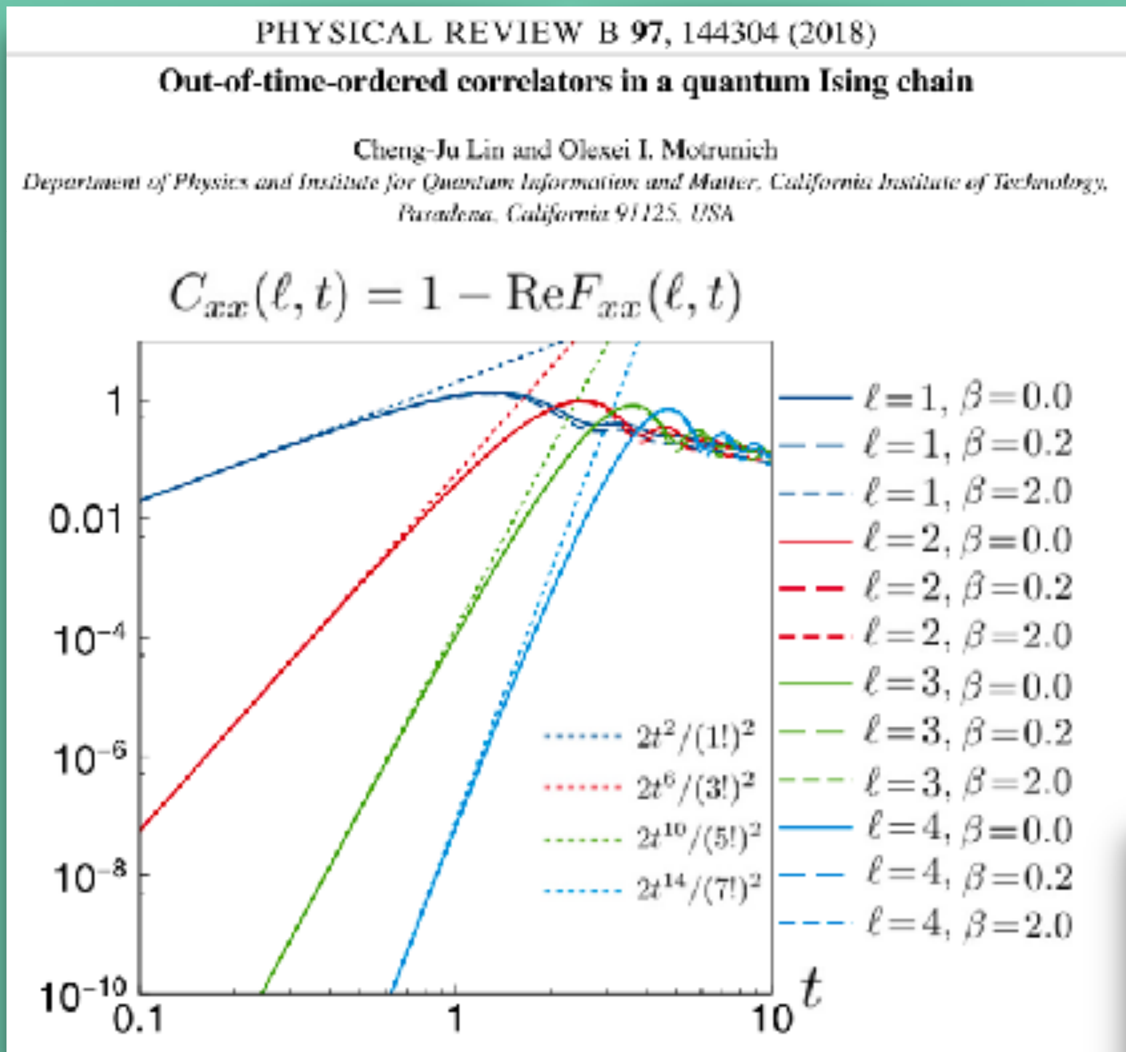


Dynamical transition from localized to uniform scrambling in locally hyperbolic systems

Mathias Steinhuber, Peter Schlagheck, Juan Diego Urbina, and Klaus Richter
Phys. Rev. E **108**, 024216 – Published 18 August 2023

$$e^{2\lambda t} \overset{?}{\Leftarrow} \text{caos}$$

no siempre

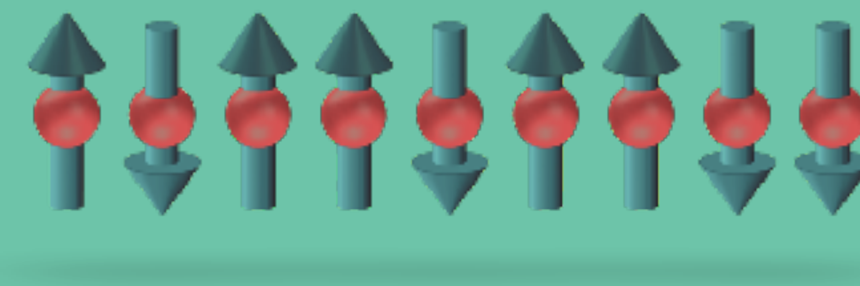


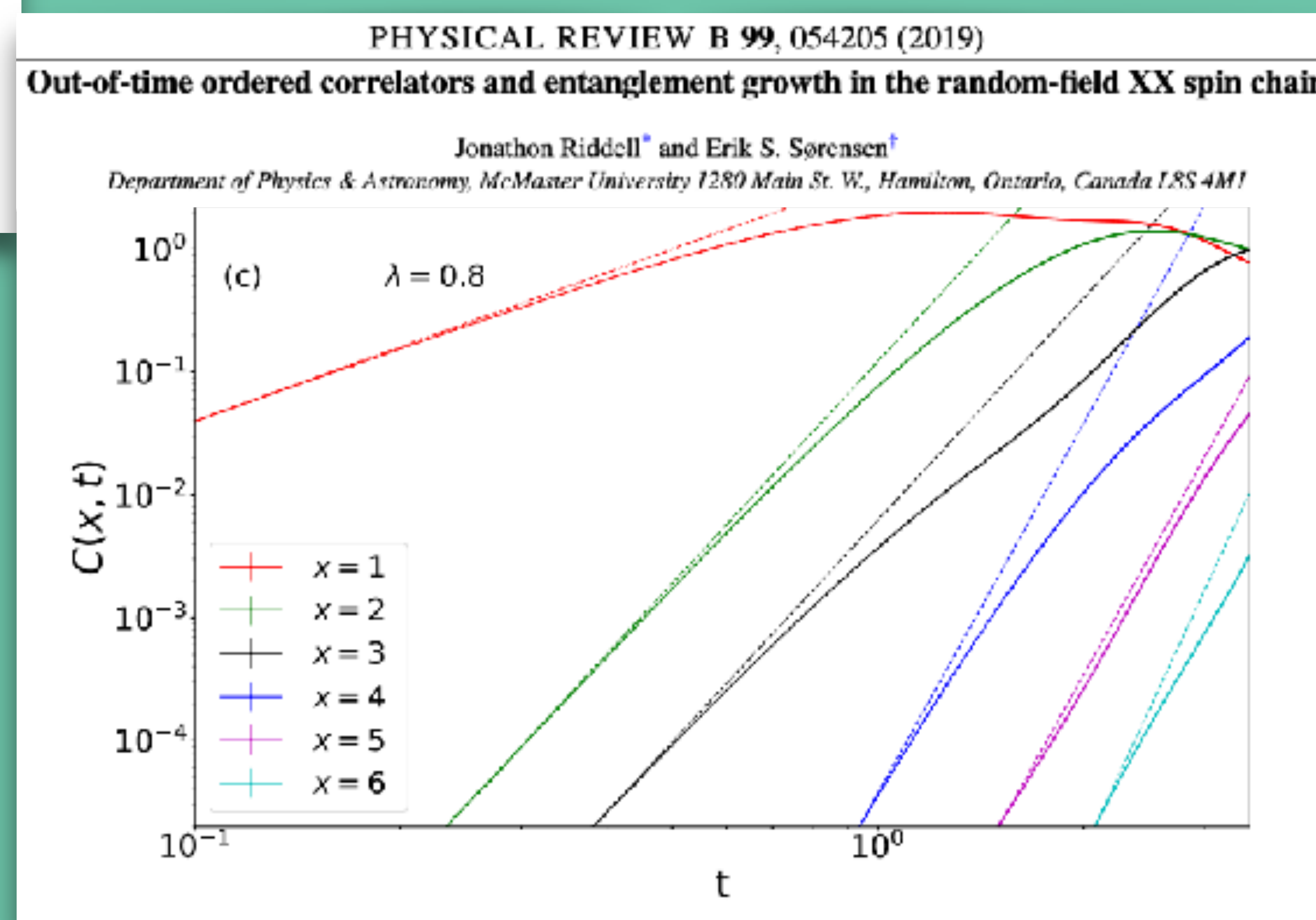
PHYSICAL REVIEW B **97**, 144304 (2018)

Out-of-time-ordered correlators in a quantum Ising chain

Cheng-Ju Lin and Olexei I. Motrunich
Department of Physics and Institute for Quantum Information and Matter, California Institute of Technology, Pasadena, California 91125, USA

$$C_{xx}(\ell, t) = 1 - \mathrm{Re}F_{xx}(\ell, t)$$



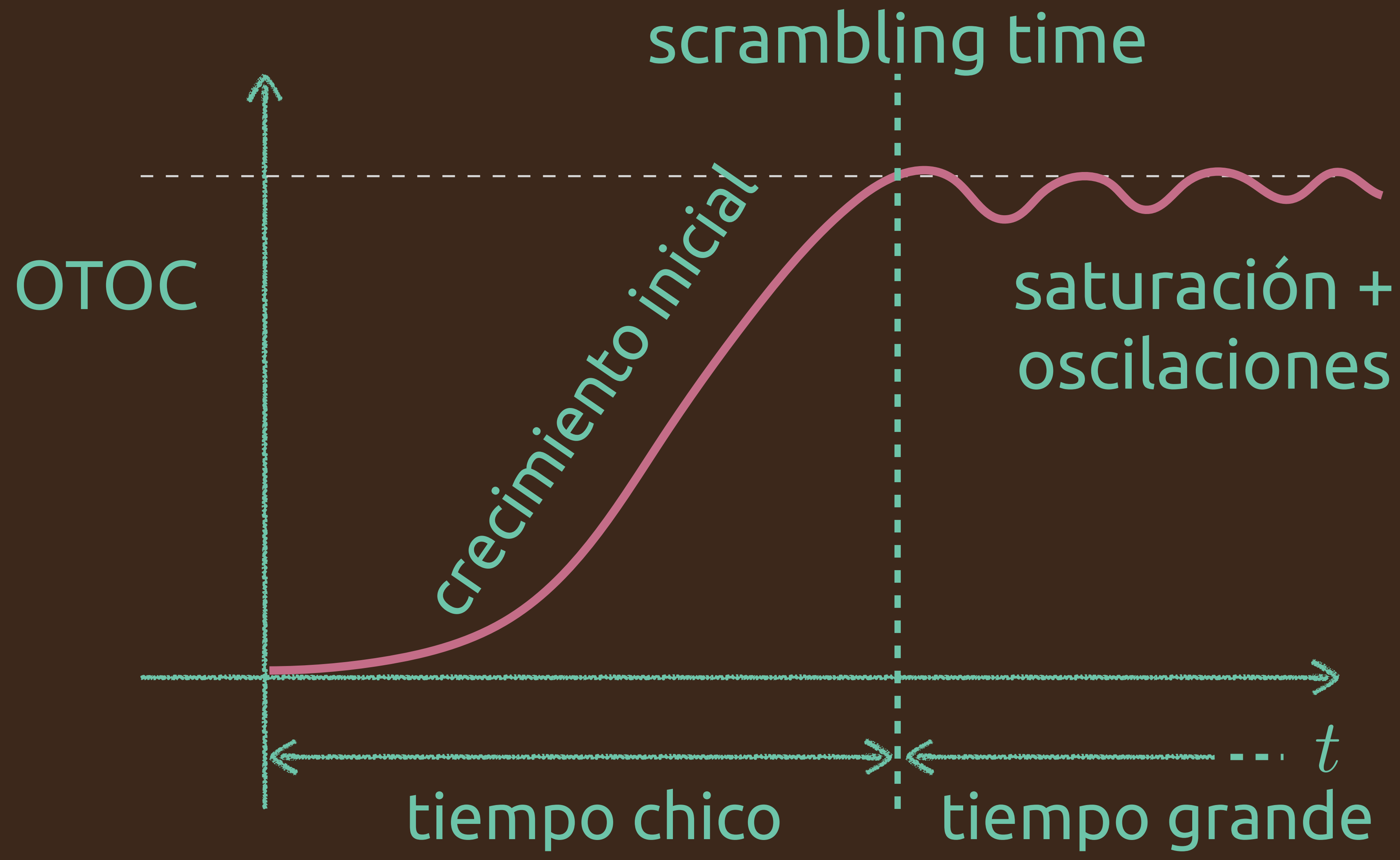


PHYSICAL REVIEW B **99**, 054205 (2019)

Out-of-time ordered correlators and entanglement growth in the random-field XX spin chain

Jonathon Riddell[*] and Erik S. Sørensen[†]
Department of Physics & Astronomy, McMaster University 1280 Main St. W., Hamilton, Ontario, Canada L8S 4M1

comportamiento genérico en sistemas acotados

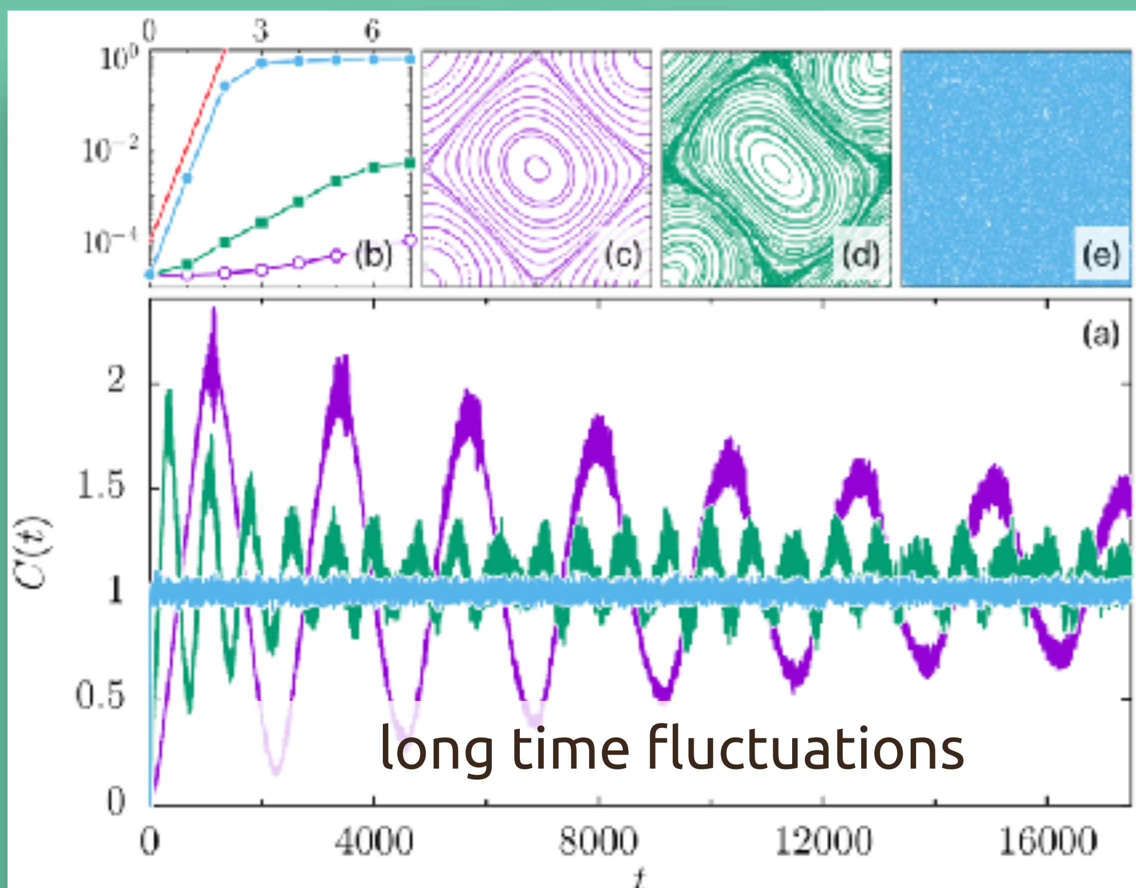


tiempos grandes

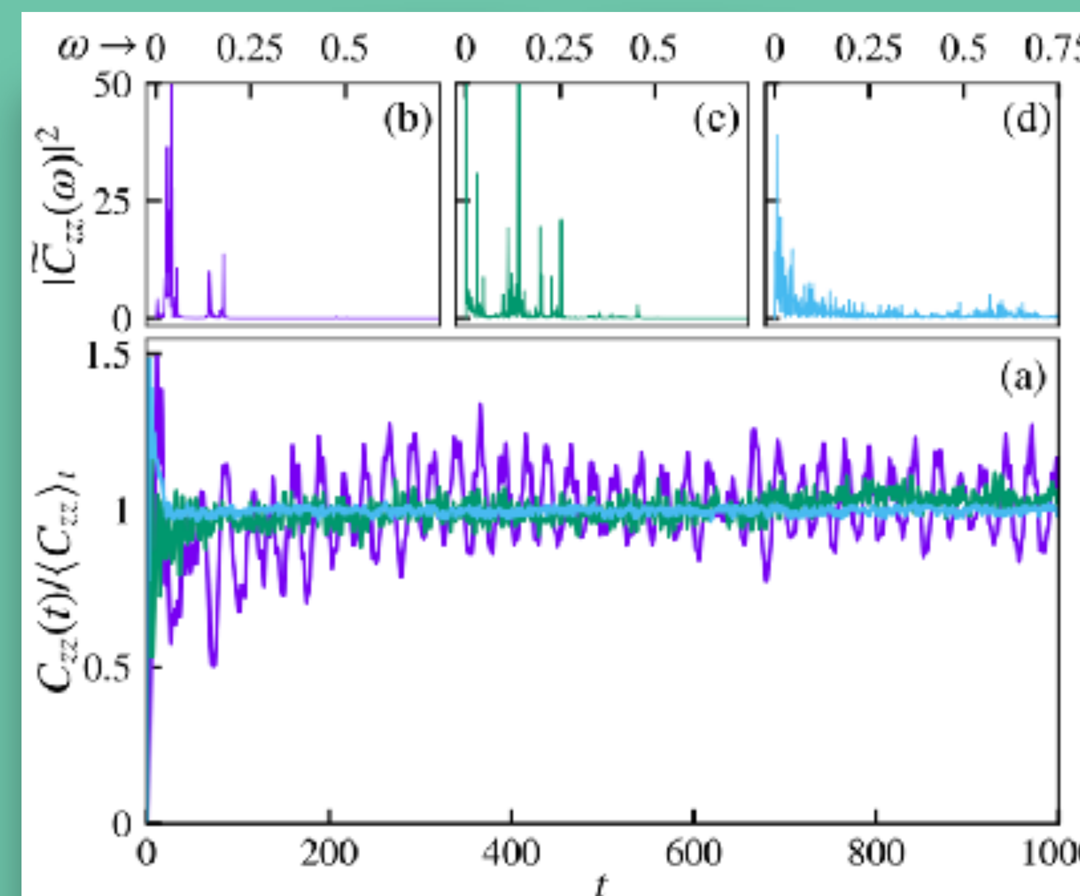
saturación

fluctuaciones

map



spin chain



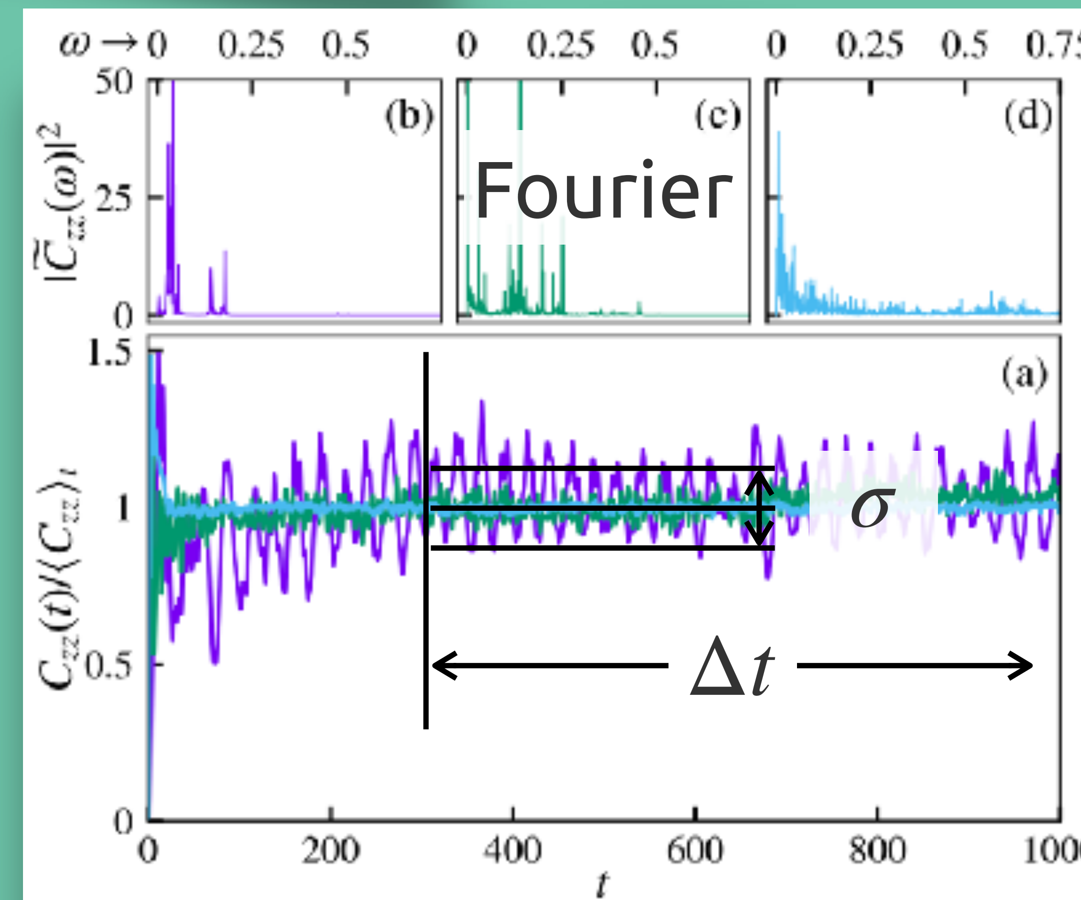
Medidas basadas en fluctuaciones a tiempos grandes del OTOC

localización en el espacio de Fourier

$$\xi_{OTOC} = \left(\int_0^\infty d\omega |\widetilde{C}(\omega)|^4 \right)^{-1}$$

varianza

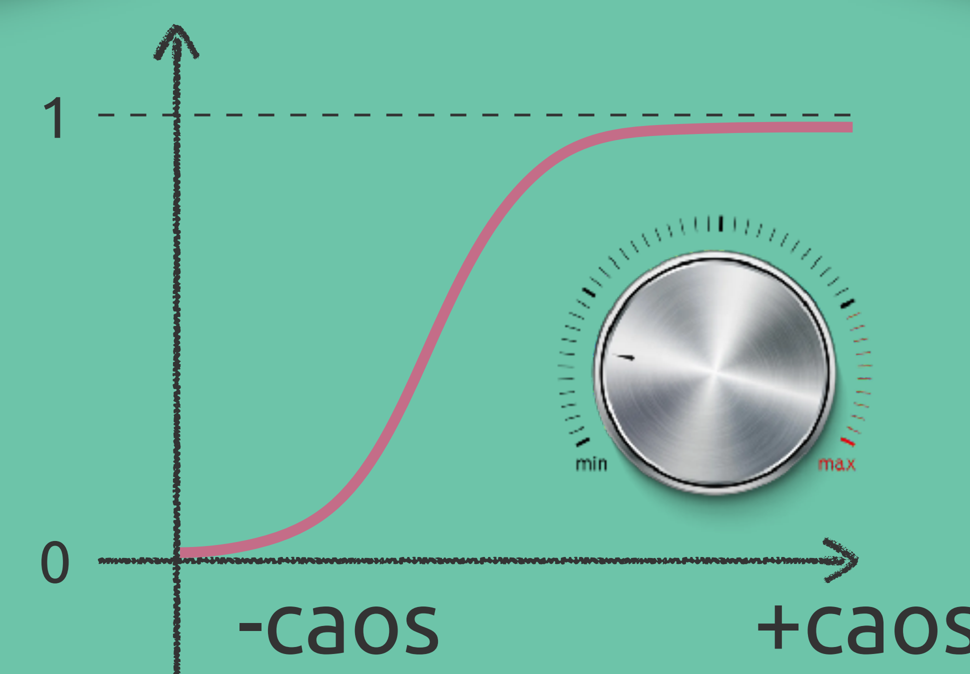
$$\sigma_{OTOC} = \sqrt{\langle C(t)^2 \rangle - \langle C(t) \rangle^2}$$



normalizadas

$$\bar{\sigma}_{OTOC}^{-1} = \sigma_{OTOC}^{-1} / (\sigma_{OTOC}^{min})^{-1}$$

$$\bar{\xi}_{OTOC} = \xi_{OTOC} / \xi_{OTOC}^{max}$$



medidas "benchmark"

Brody

$$P_B(s) = (\beta+1)\, b\, s^{\beta} e^{-b\, s^{\beta+1}}, \quad b = \left[\Gamma\left(\frac{\beta+2}{\beta+1}\right)\right]^{\beta+1}$$

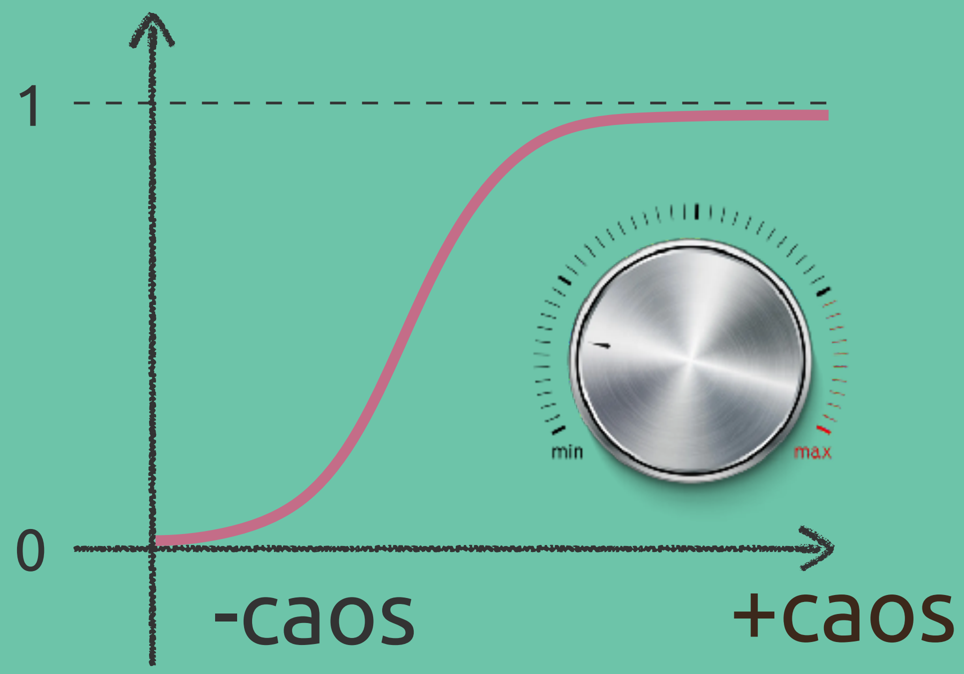
$\beta \to 1$ Wigner-Dyson

$\beta \to 0$ Poisson

IPR (normalizado) de autoestados

$|\psi_i\rangle = \sum_j a_{ij} |\phi_j\rangle$. depende de la base

$$\xi_E(i) = \left(\sum_{j=0}^{D-1} |a_{ij}|^4\right)^{-1}$$



Berry-Robnick

Berry & Robnick JPA 17, 2413 (1984)

$$P_{\rm BR}(s) = \left[2(1-\bar\rho)\bar\rho + \frac{\pi}{2}\bar\rho^3 s\right] e^{-(1-\bar\rho)s - \frac{\pi}{4}\bar\rho^2 s^2} + (1-\bar\rho)^2 \,\mathrm{erfc}\left(\frac{\sqrt{\pi}}{2}\bar\rho s\right) e^{-(1-\bar\rho)s}$$

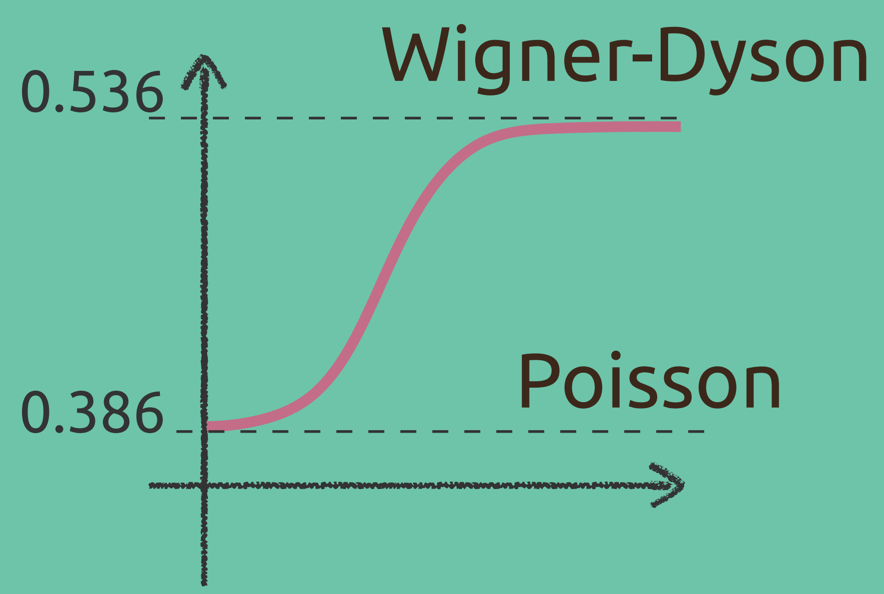
$\bar\rho^2$ "$\equiv$" β

Spacing ratios

Oganesyan & Huse PRB 75 155111 (2007)
Atas, Y. Y., Bogomolny, E., Giraud, O., & Roux. PRL, 110, 084101 (2013).

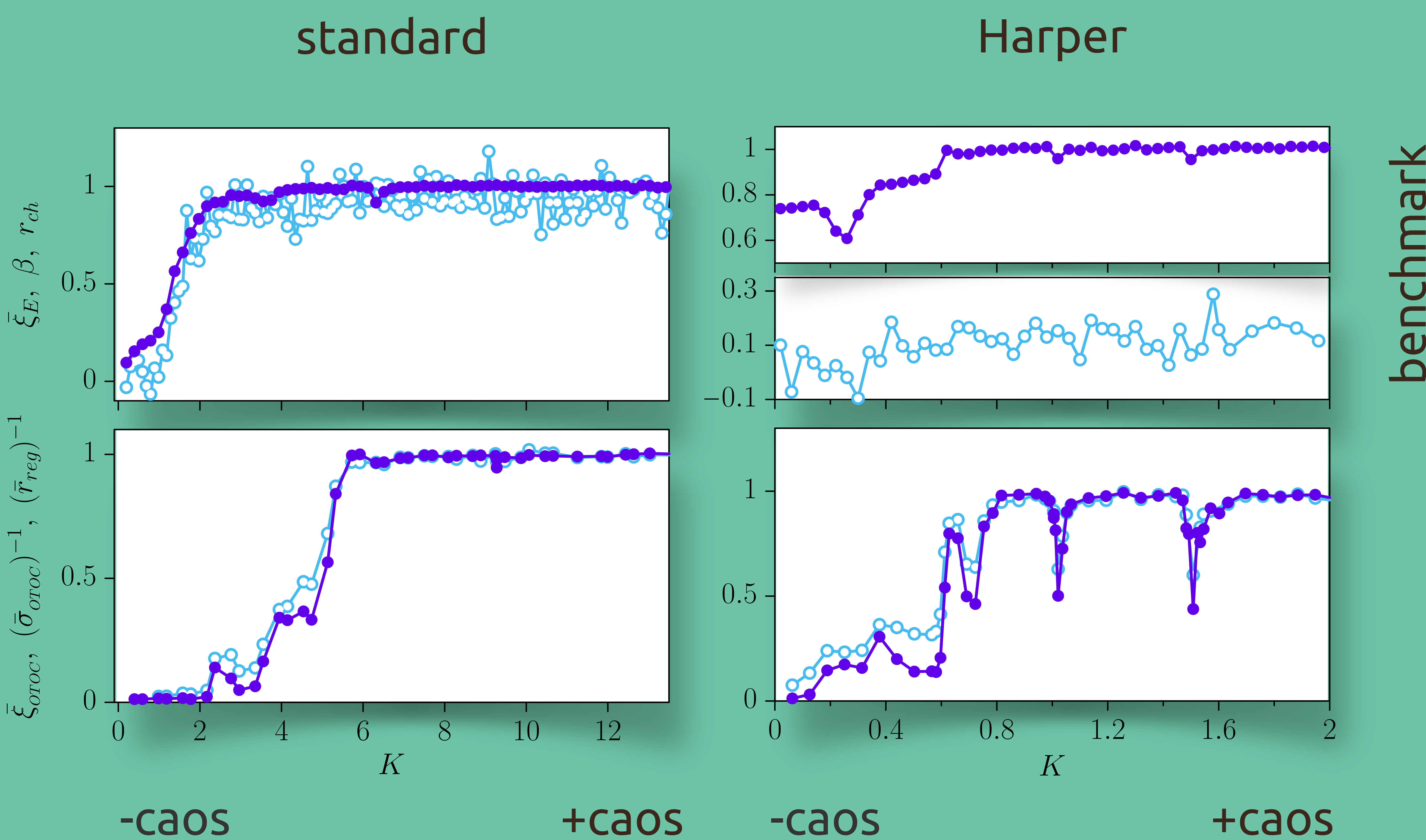
$$r_n = s_{n+1}/s_n \qquad s_n = E_{n+1} - E_n$$

$$\bar r = \min(r_n, 1/r_n)$$



no unfolding
no need of full spectrum

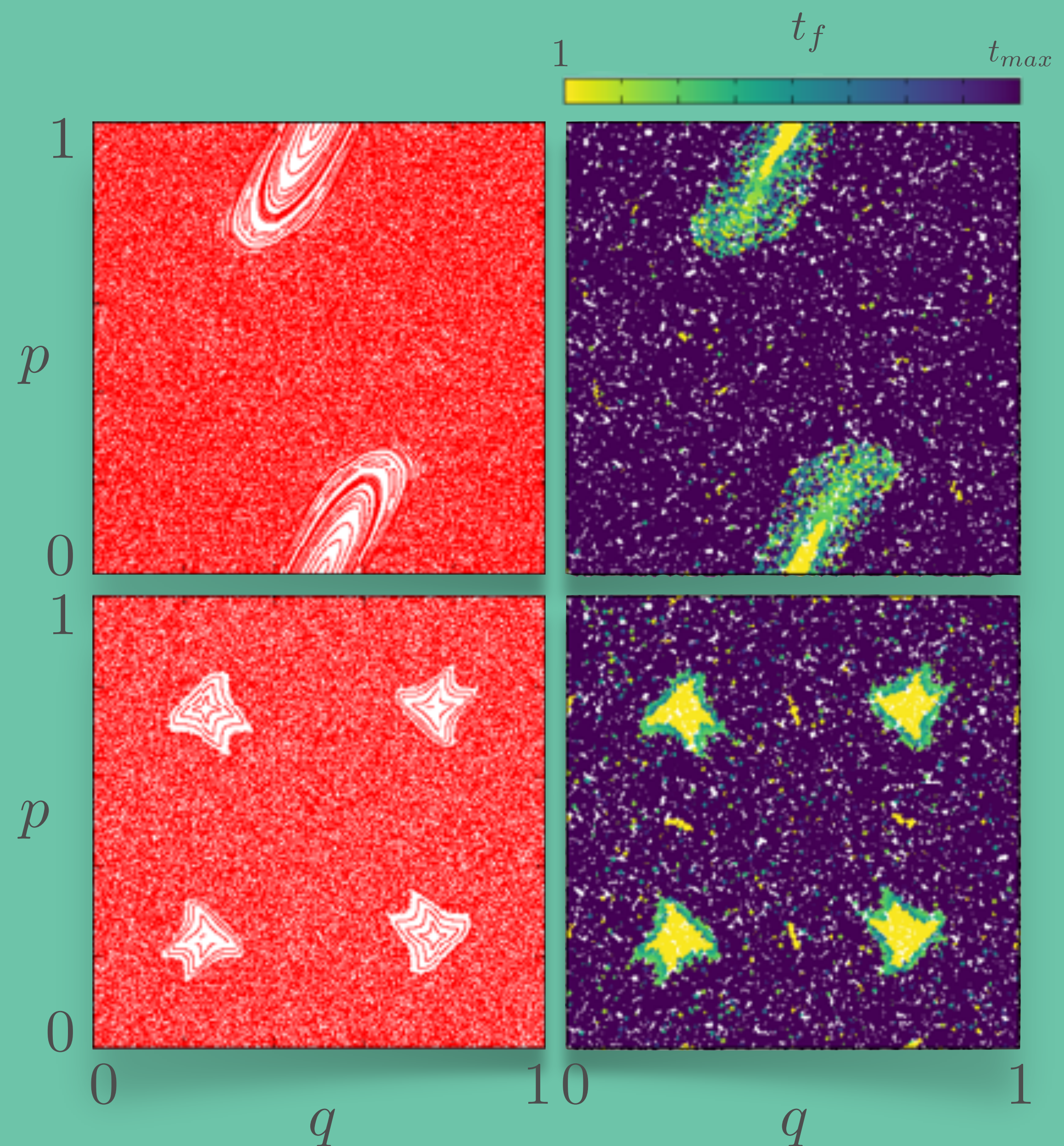
mapas cuánticos



standard
Harper
benchmark
$\bar{\xi}_E,\ \beta,\ r_{ch}$
$\bar{\xi}_{OTOC},\ (\bar{\sigma}_{OTOC})^{-1},\ (\bar{r}_{reg})^{-1}$
K
K
-caos
+caos
-caos
+caos

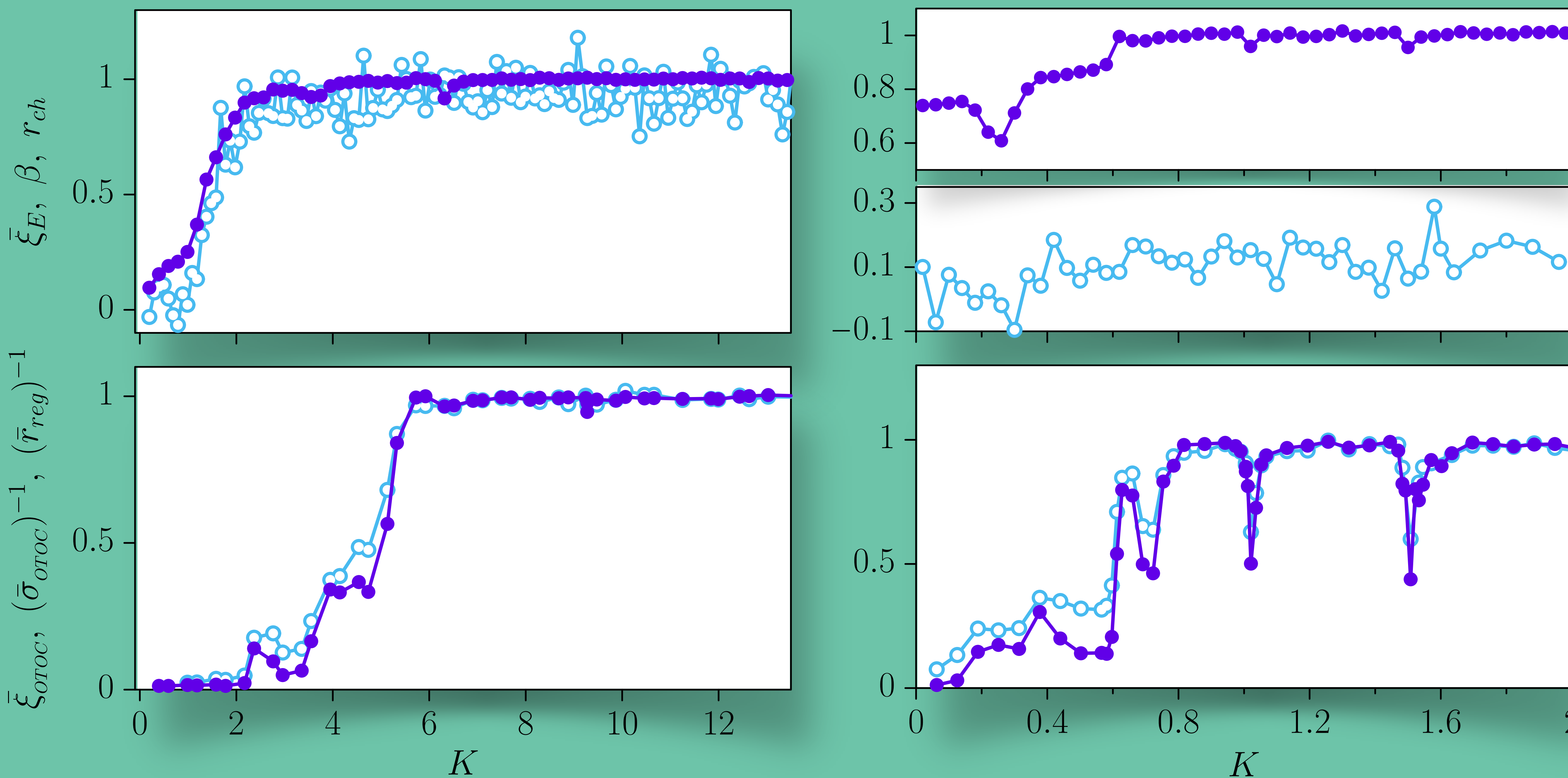
estimate chaotic & regular areas

- Random n_{tot} initial conditions $(q_i^{(0)}, p_i^{(0)})$
- Evolve map up to t_{max}
- If $||(q_i^{(t)}, p_i^{(t)}) - (q_i^{(0)}, p_i^{(0)})|| < d$ then stop.
- Record t_f
- Else stop at t_{max}
- $r_{ch} = \frac{n_{t_{max}}}{n_{tot}}$ $\approx$ size of chaotic region
- $r_{reg} = 1 - r_{ch} \approx$ size of regular region



standard map

Harper map



-caos

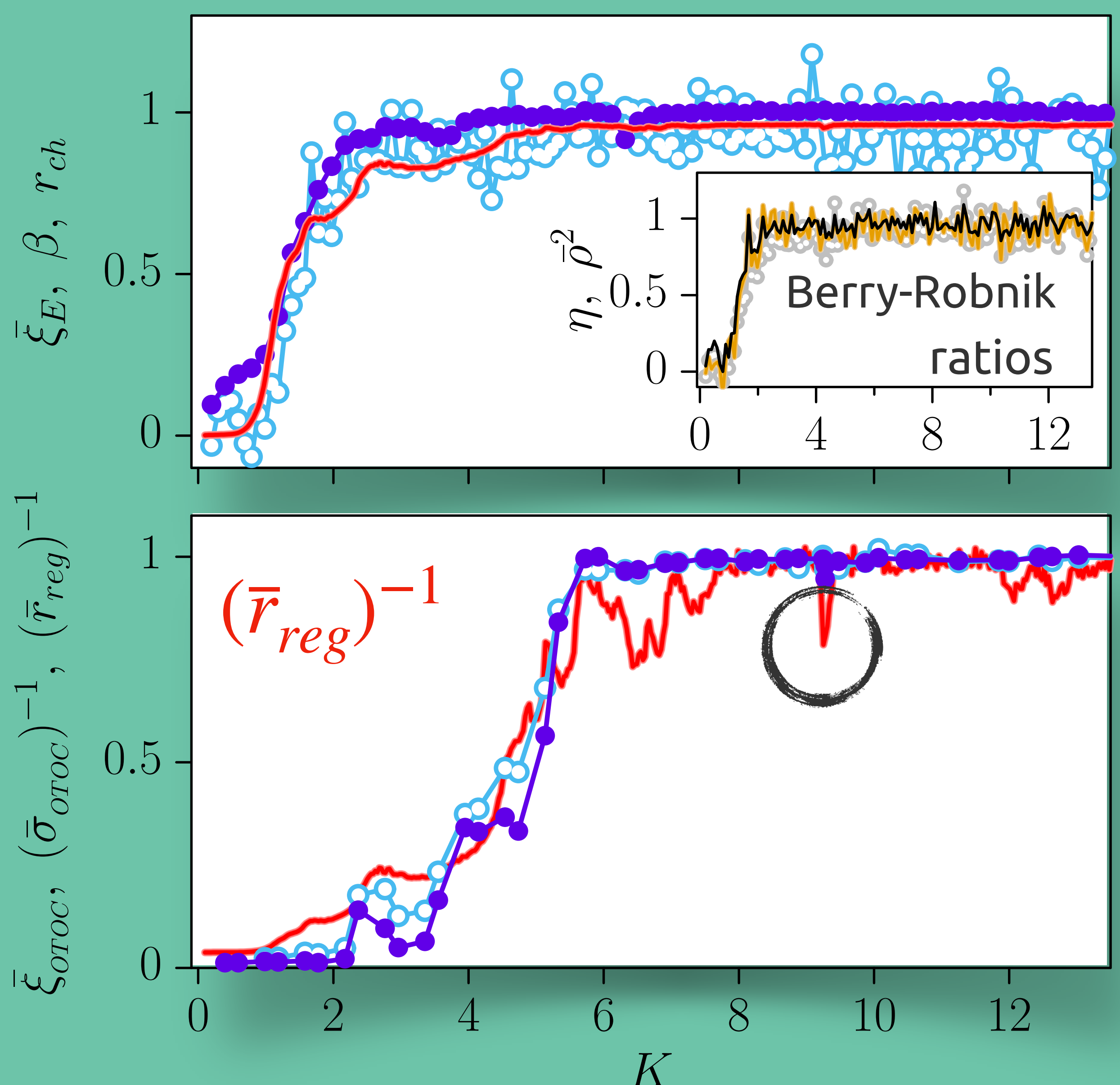
+caos

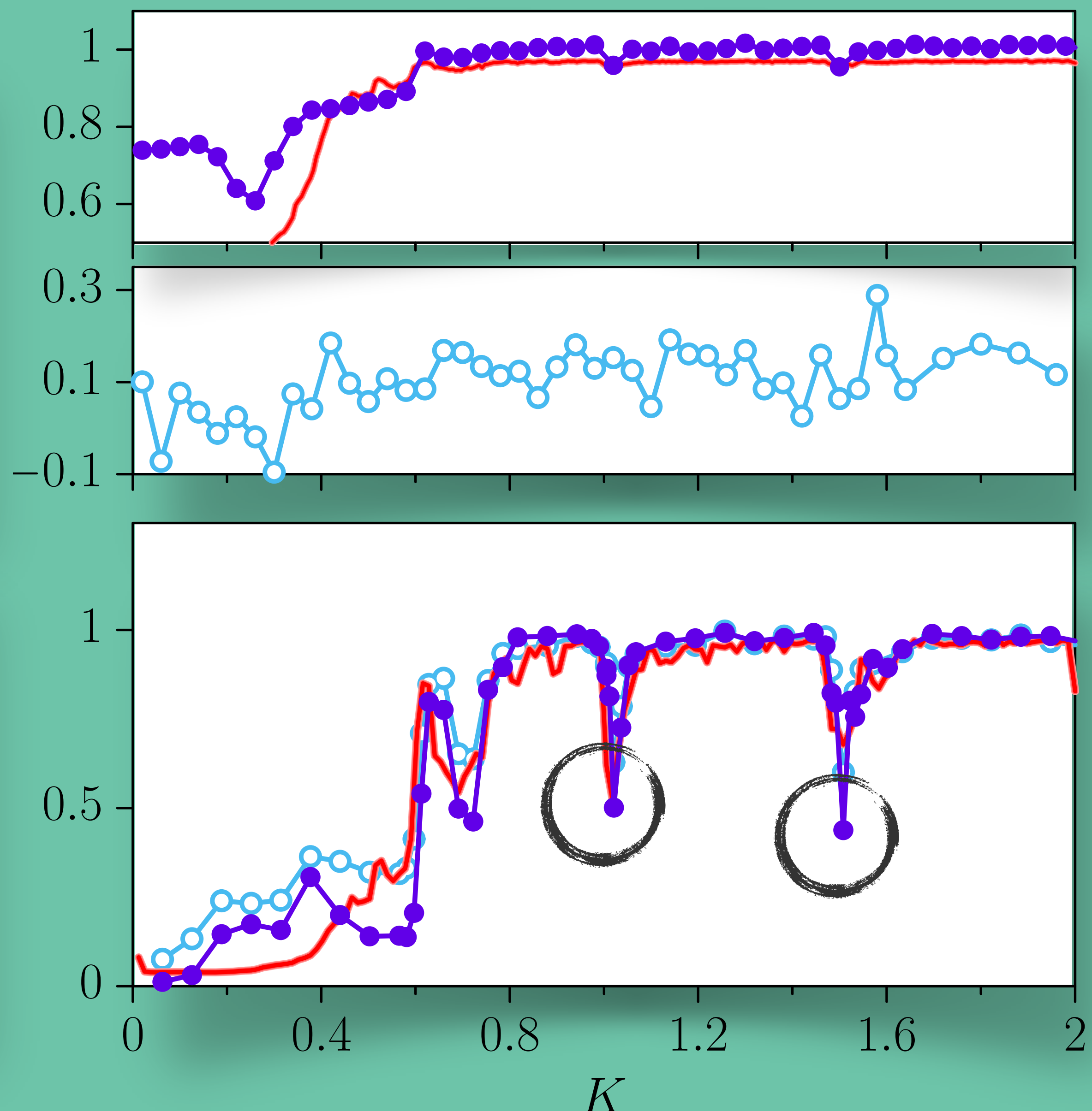
-caos

+caos

mapa estandar

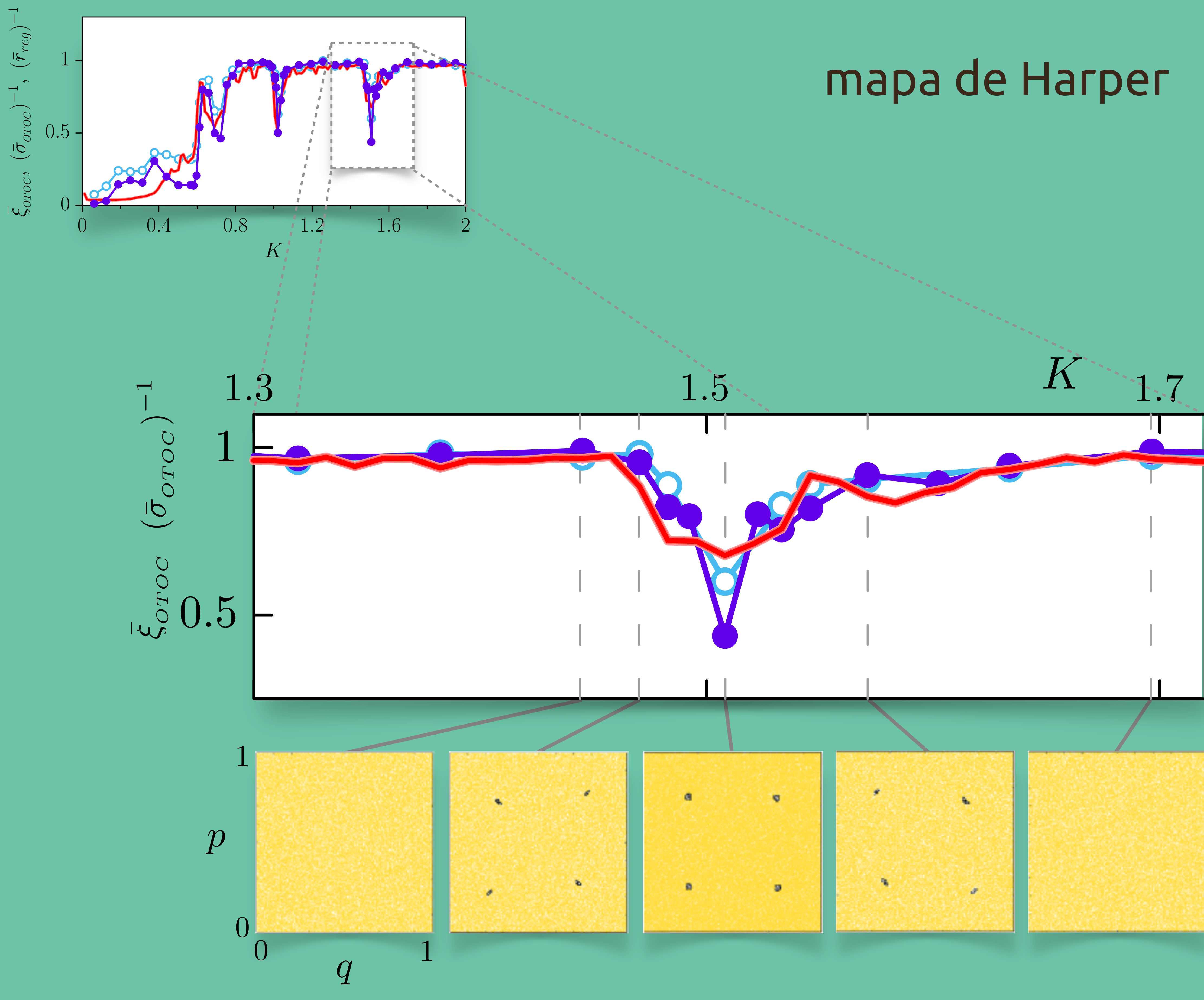
mapa de Harper





-caos +caos -caos +caos

- $r_{reg} = 1 - r_{ch} \approx$ size of regular region



mapa de Harper
$\bar{\xi}_{OTOC}$, $(\bar{\sigma}_{OTOC})^{-1}$, $(\bar{r}_{reg})^{-1}$
1
0.5
0
0
0.4
0.8
1.2
1.6
2
K
1.3
1.5
K
1.7
$\bar{\xi}_{OTOC}$ $(\bar{\sigma}_{OTOC})^{-1}$
1
0.5
1
p
0
0
q
1

cadenas de spin

L spins $1/2$

$\mu = x, y, z$

$\hat{S}_i^{\mu} = \frac{1}{2}\hat{\sigma}_i^{\mu}$ spin operator

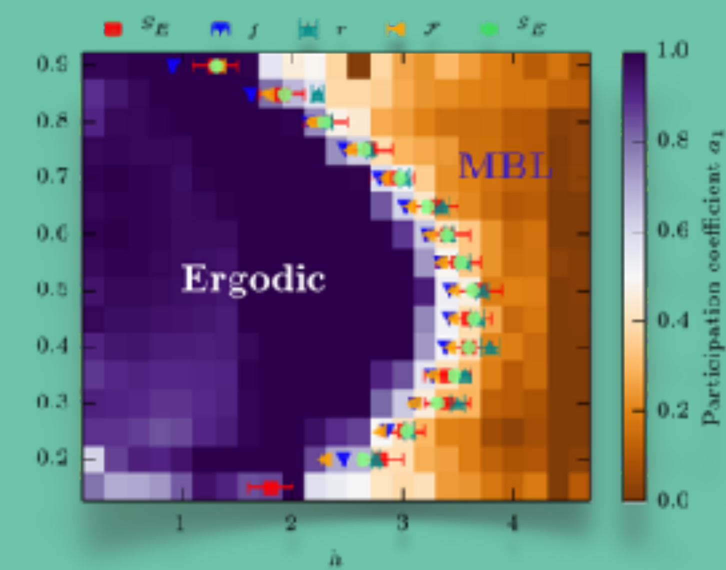
site $i = 0,1\ldots, L-1$

$$C_{\mu\nu}(l,t) = \frac{1}{2}\langle[\hat{\sigma}_0^{\mu}(t), \hat{\sigma}_l^{\nu}]^2\rangle$$
$$= 1 - Re\left\{Tr[\hat{\sigma}_0^{\mu}(t)\hat{\sigma}_l^{\nu}\hat{\sigma}_0^{\mu}(t)\hat{\sigma}_l^{\nu}]\right\}/D$$

D : Hilbert space dimension

cadena de Heisenberg con un campo aleatorio

$$\hat{H} = \sum_{i=0}^{L-2} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \hat{S}_i^z \hat{S}_{i+1}^z) + \sum_{i=0}^{L-1} h_i \hat{S}_i^z$$



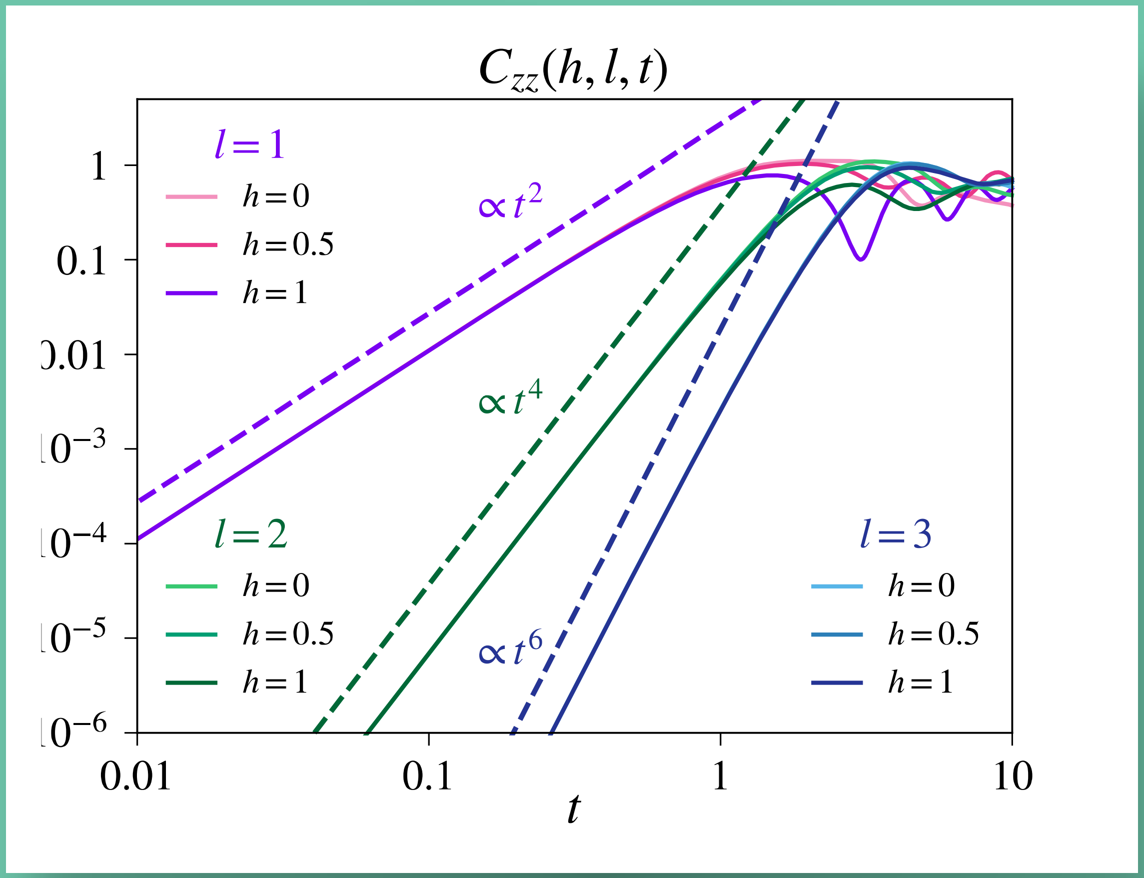
estudios de transición MBL

(review @ Alet & Laflorencie, C. R. Physique 19 (2018) 498–525)

Luitz, Laflorencie & Alet
PRB 91, 081103(R) (2015)

t chico

$$C_{zz}(l,t) \approx \frac{1}{2} \frac{t^{2l}}{(2l!)^2}$$



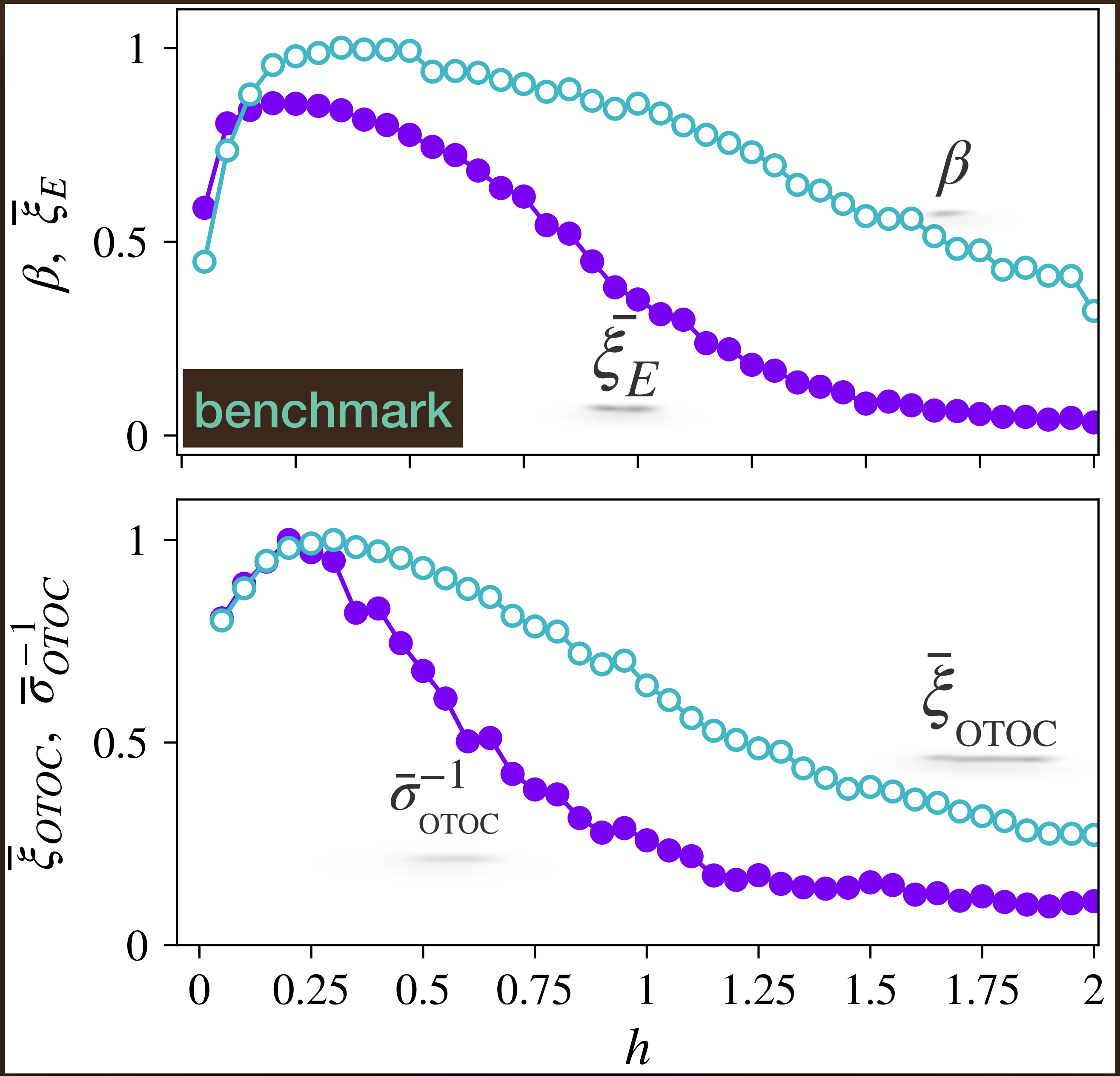
$\bar{\xi}_E$ spin site basis
10% center of the spectrum

Symmetry subspaces of dim

$$D_N \equiv \dim\left(\hat{\mathcal{S}}_N\right) = \binom{L}{N} = \frac{L!}{N!(L-N)!}$$

promedio sobre > 100 realizaciones

chain of length $L = 13$ with
$N = 5$ fixed spins up (or down)
$D = 1287$



chain of length $L = 9$ with
$N = 5$

E. Fortes, I.G-M., R. Jalabert & D.A. Wisniacki, PRE **100**, 042201 (2019)

Resultados equivalentes para

$$\hat{H}(\lambda) = \hat{H}_0 + \lambda \hat{H}_1$$

Modelo XXZ perturbado

$$\hat{H}_0 = \sum_{i=0}^{L-2} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \mu \hat{S}_i^z \hat{S}_{i+1}^z),$$

$$\hat{H}_1 = \sum_{i=0}^{L-3} (\hat{S}_i^x \hat{S}_{i+2}^x + \hat{S}_i^y \hat{S}_{i+2}^y + \mu \hat{S}_i^z \hat{S}_{i+2}^z).$$

Next Nearest Neighbor

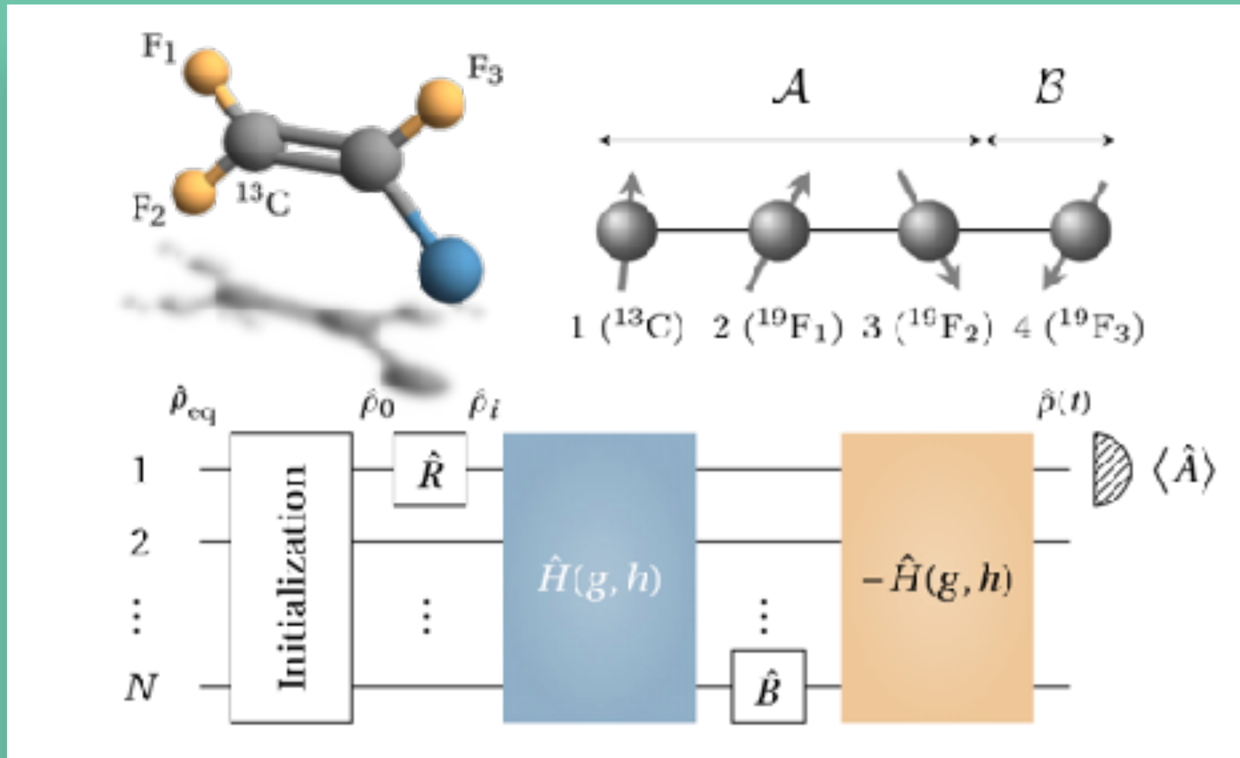
Modelo de Ising con campo magnético inclinado

$$\hat{H} = J \sum_{i=0}^{L-2} \hat{S}_i^z \hat{S}_{i+1}^z + B \sum_{i=0}^{L-1} \left(sin\,(\theta)\, \hat{S}_i^x + cos\,(\theta)\, \hat{S}_i^z \right)$$

E. Fortes, I.G-M., R. Jalabert & D.A. Wisniacki, PRE **100**, 042201 (2019)

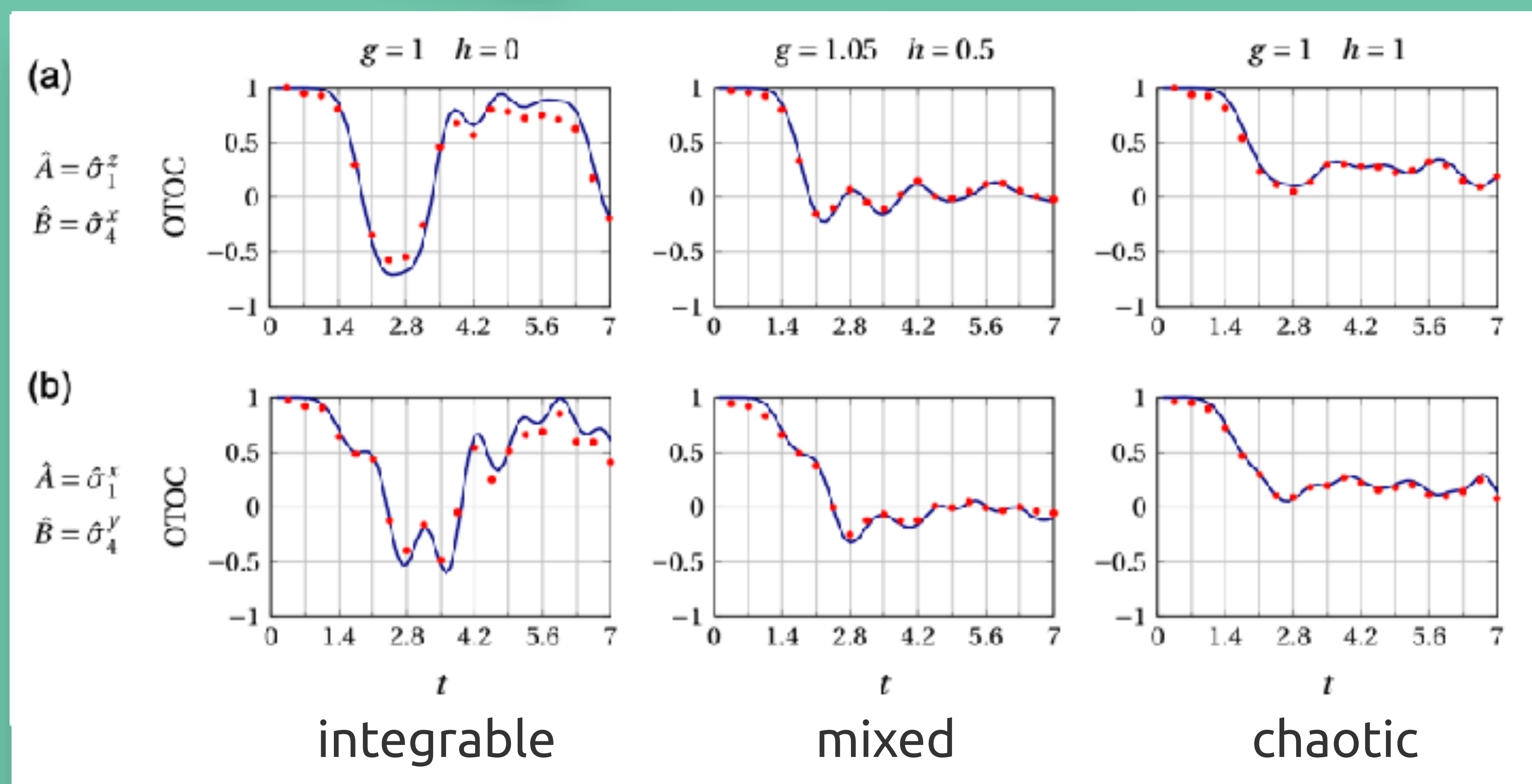
OTOC medido en una simulacion de una cadena de Ising en NMR

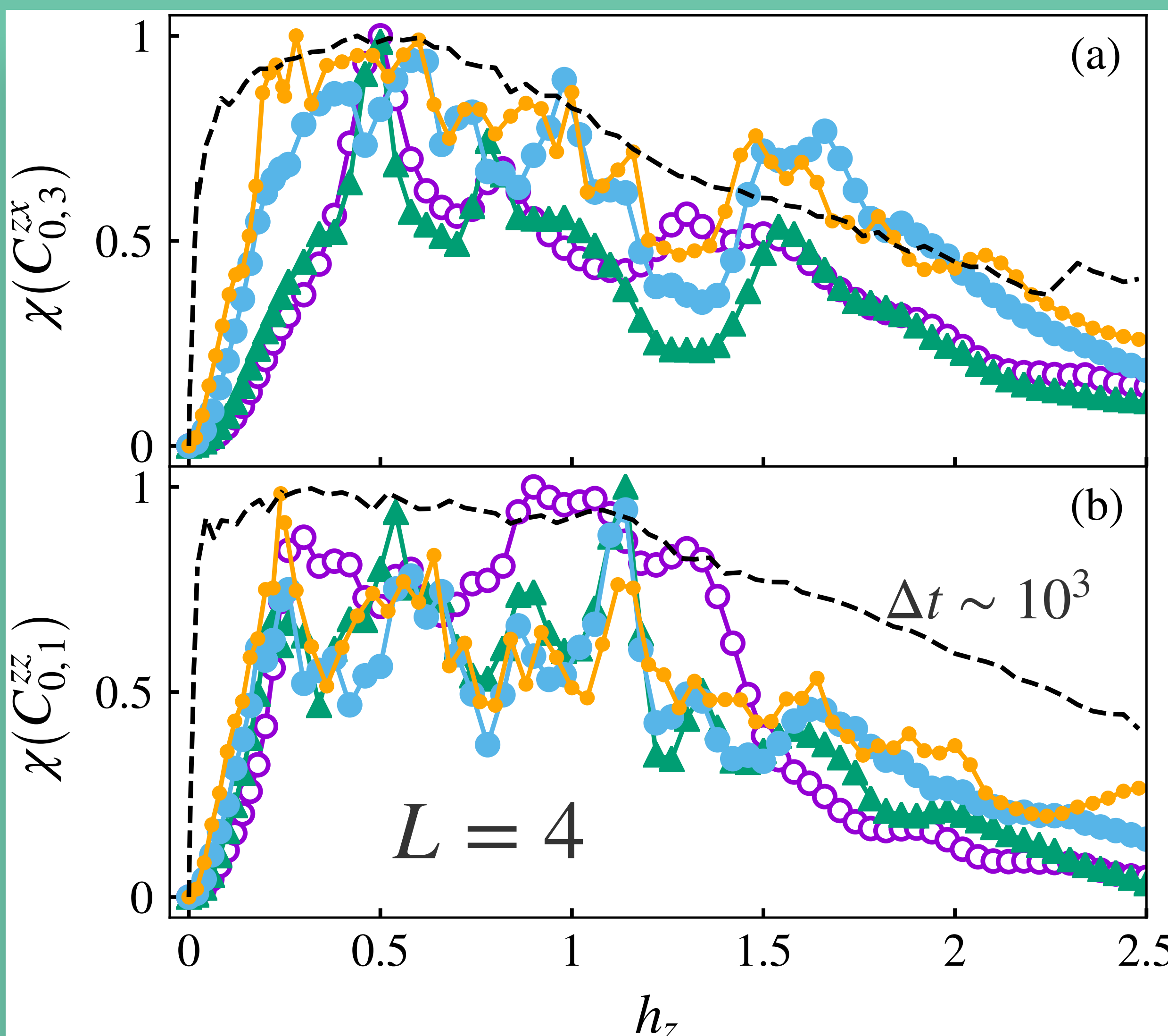
Li et al., PRX **7**, 031011 (2017)



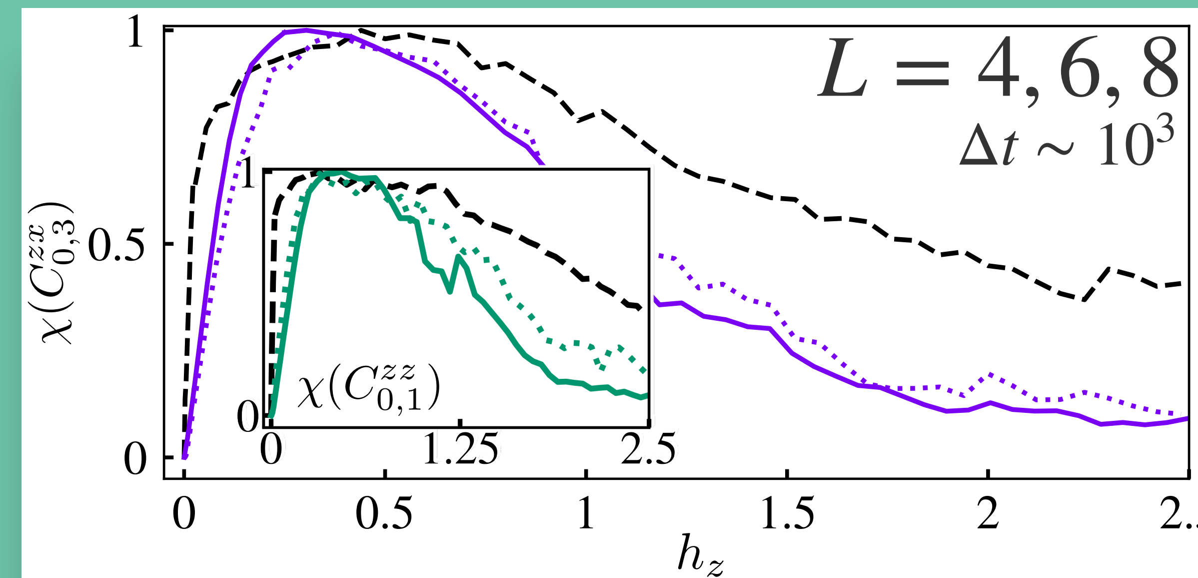
$$\hat{H} = \sum_i \left(-\hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + g\hat{\sigma}_i^x + h\hat{\sigma}_i^z\right)$$

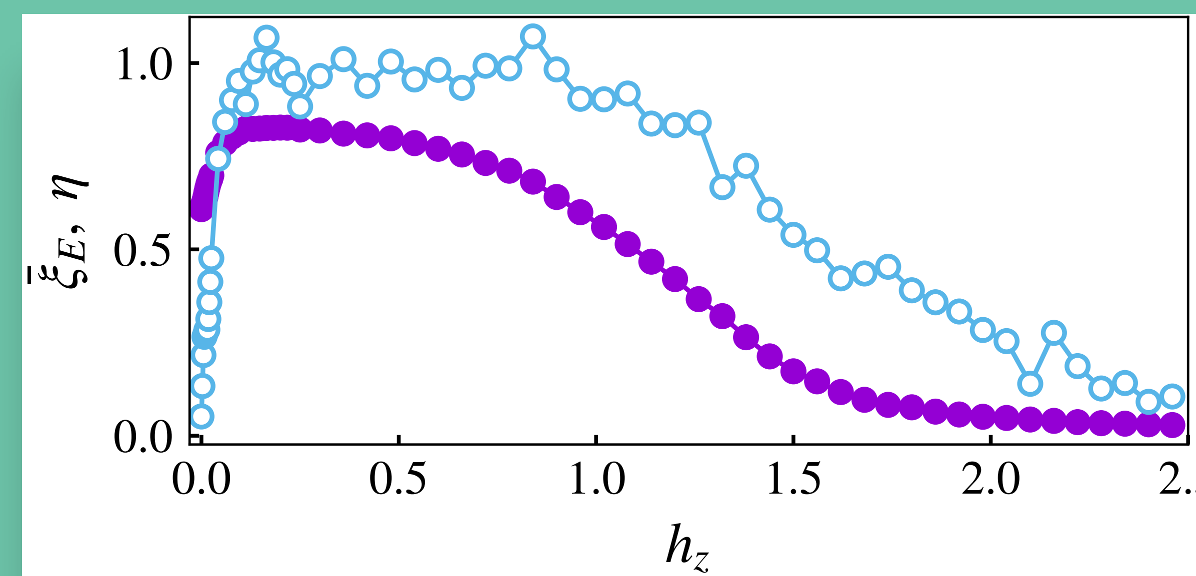
Cadenas chicas tiempo corto





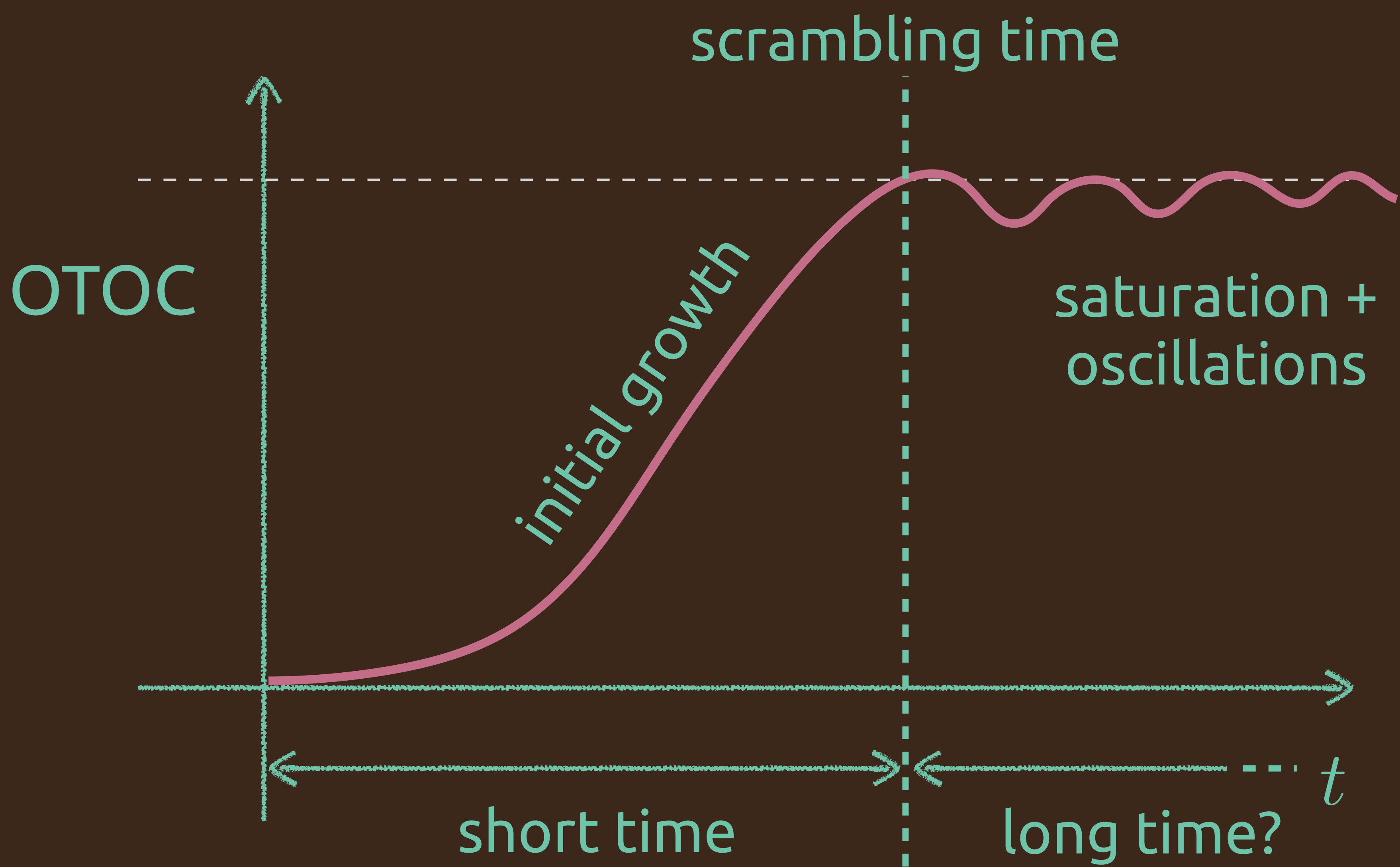
time windows $\Delta t = 5.5, 10, 20, 40$

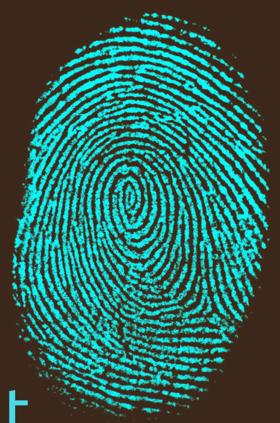




E.M. Fortes, IG-M, R.A. Jalabert, D. A. Wisniacki. EPL, **130** (2020) 60001.

Summary





fast scramblers
(q. maps, black holes, SYK) : Lyapunov

Slow scramblers
(spin chains) : power law

chaos fingerprint
in both slow and fast
scramblers

One more thing

tiempo “intermedio”

most works have focused on the initial times

initial operator spreading

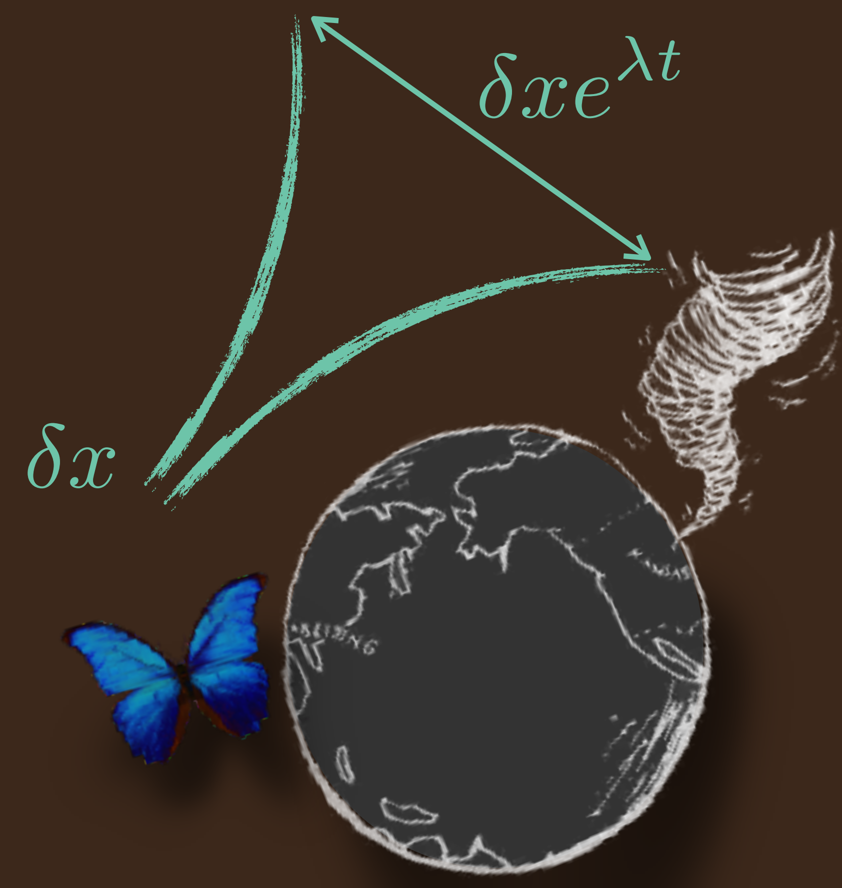
scrambling

butterfly

effect

Caos clásico: 2 características

separación exponencial



mixing

Chaos in the black hole S-matrix

Joseph Polchinski

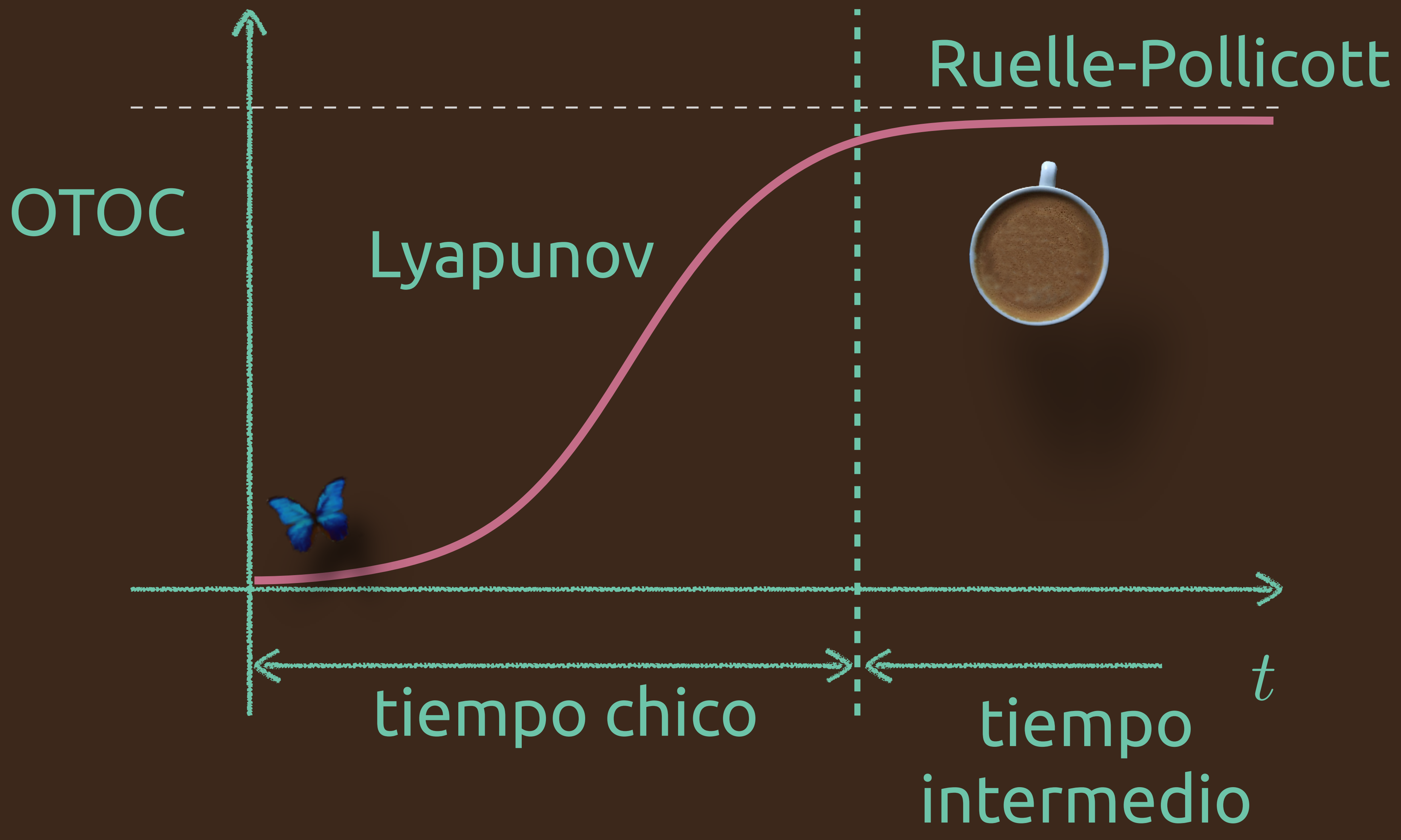
Kavli Institute for Theoretical Physics
University of California
Santa Barbara, CA 93106-4030

arXiv:1505.08108v2 [hep-th] 1 Jun 2015

Figure 2: Commutator-squared versus time. The exponential Lyapunov growth and Ruelle decay are noted.

Caos

OTOC debería tener los dos (Polchinsky 2015)



resonancias de Ruelle-Policott

autovalores aislados del operador de Koopman-Perron-Frobenius

Blank, Keller & Liverani Nonlinearity 15 (2002)

Determinan el decaimiento de las correlaciones

INSTITUTE OF PHYSICS PUBLISHING NONLINEARITY

Nonlinearity **16** (2003) 1685–1713

Spectral properties of noisy classical and quantum propagators

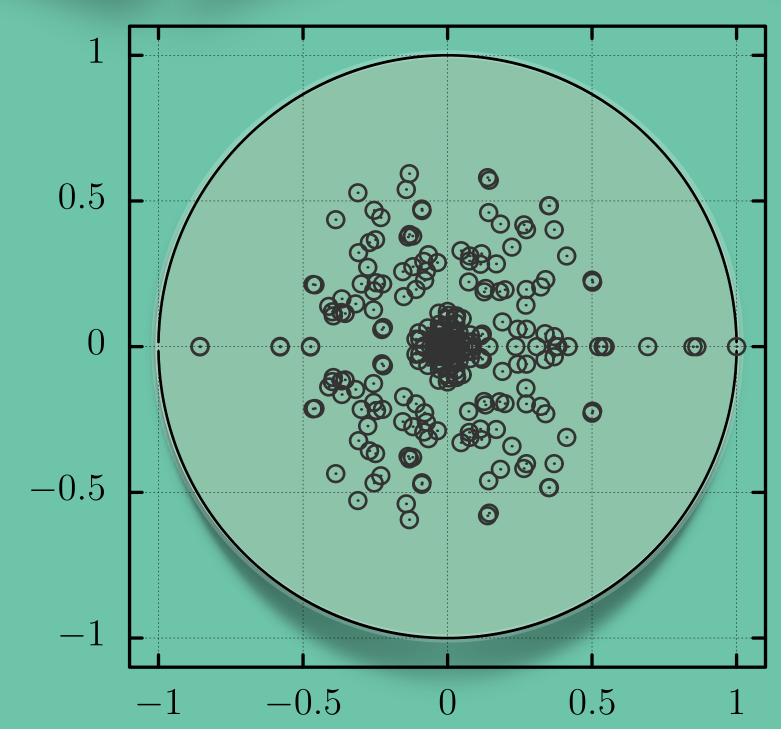
Stéphane Nonnenmacher

quantum coarse grained → classical coarse grained → Ruelle-Pollicott

el orden de los límites es importante

$$N \to \infty$$

coarse graining $\to 0$



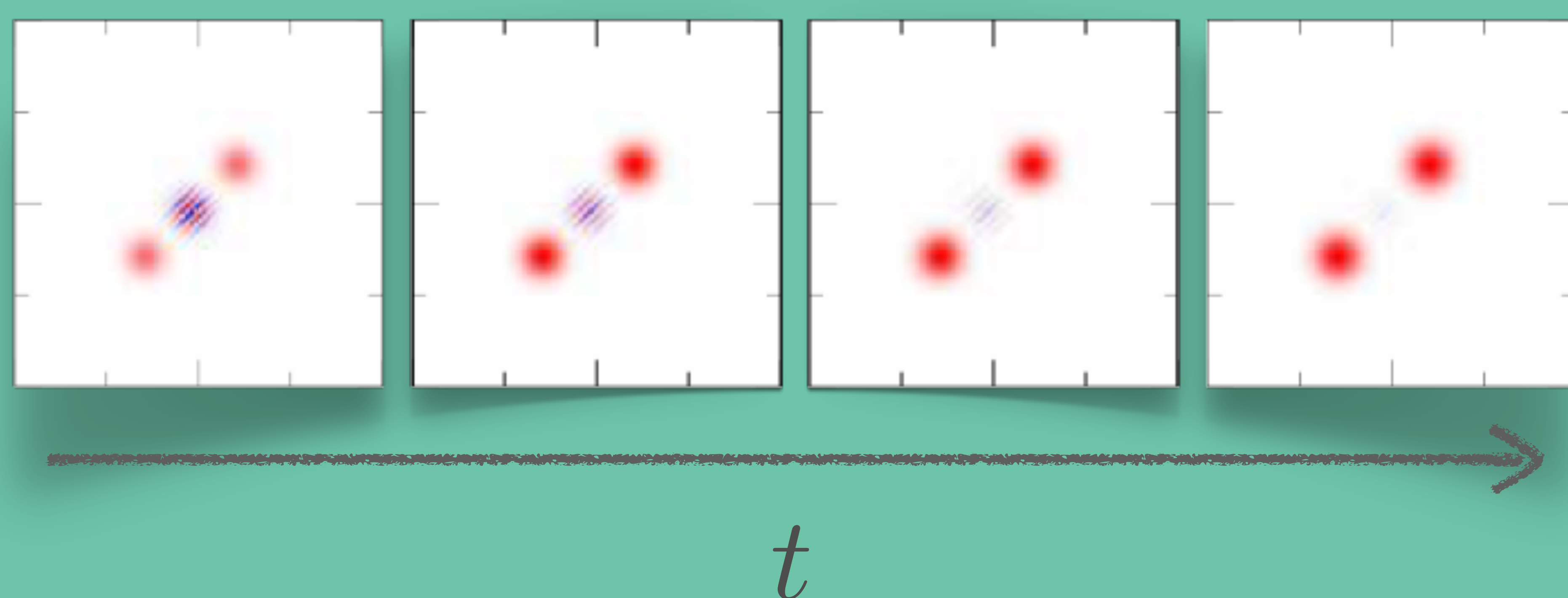
$$\hat{A}_{t+1} = \mathbf{S}_\epsilon(\hat{A}_t) \equiv \mathbf{D}_\epsilon(\hat{U}^\dagger \hat{A} \hat{U})$$

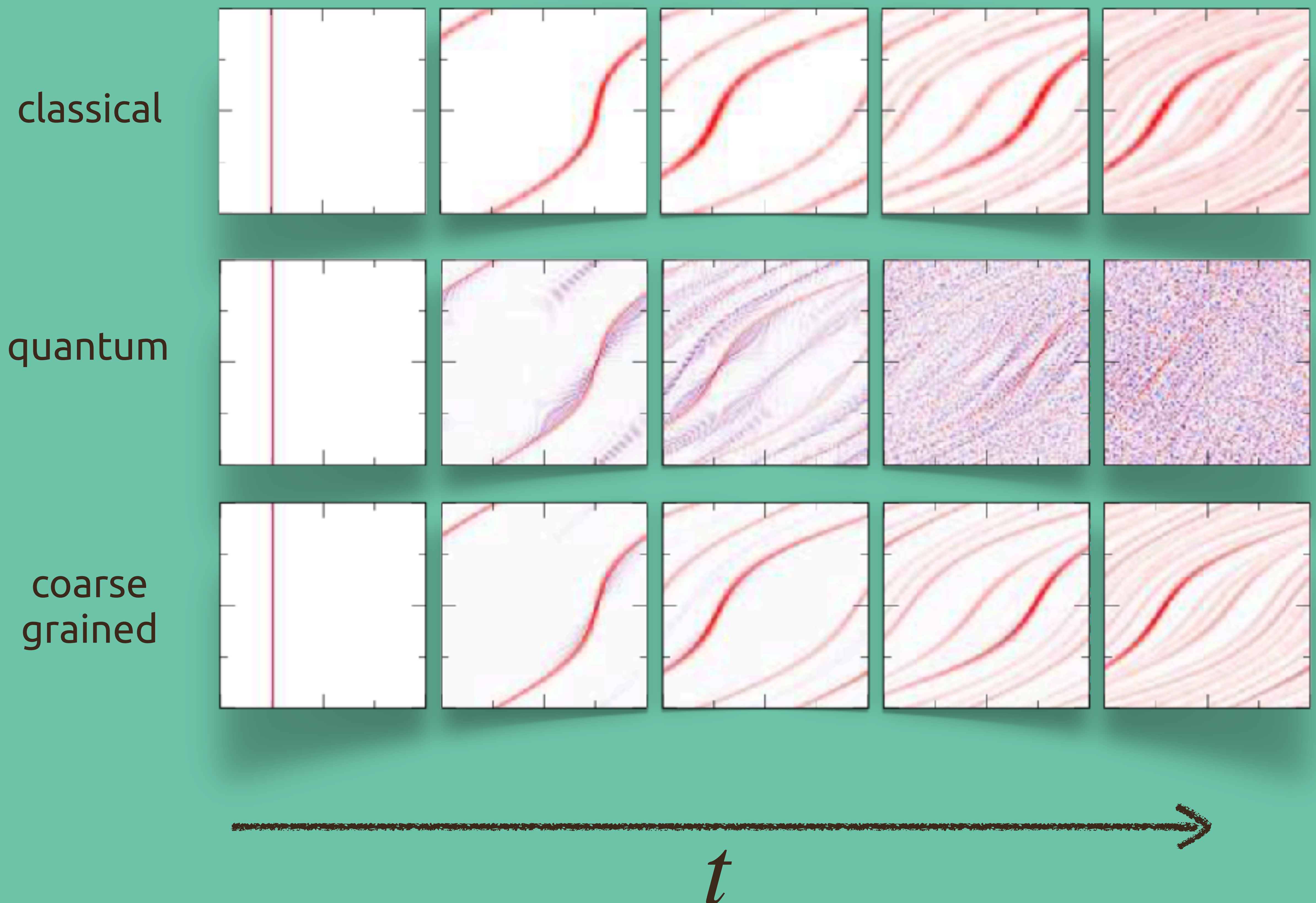
$$\mathbf{D}_e(\hat{O}) \stackrel{\text{def}}{=} \sum_\xi c_\epsilon \hat{T}_\xi^\dagger \hat{O} \hat{T}_\xi$$

Gaussian kernel

coarse graining

$$\hat{A}_{t+1} = \sum_\xi c_\epsilon(\xi) \hat{T}_\xi^\dagger \hat{U}^\dagger \hat{A}_t \hat{U} \hat{T}_\xi$$

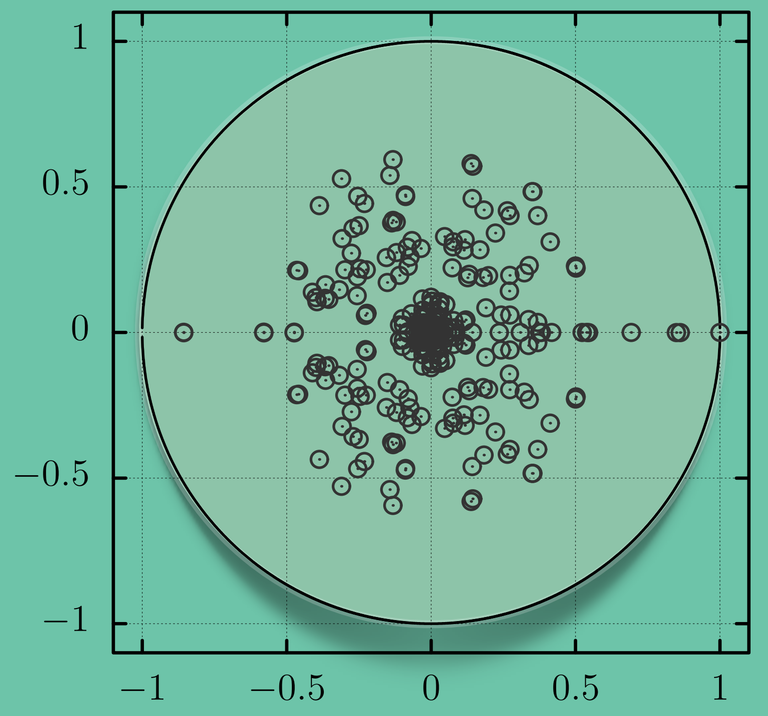



classical
quantum
coarse grained
t

$\hbar \sim 1/N$ matriz $N^2 \times N^2$

iterar (tipo Krylov)

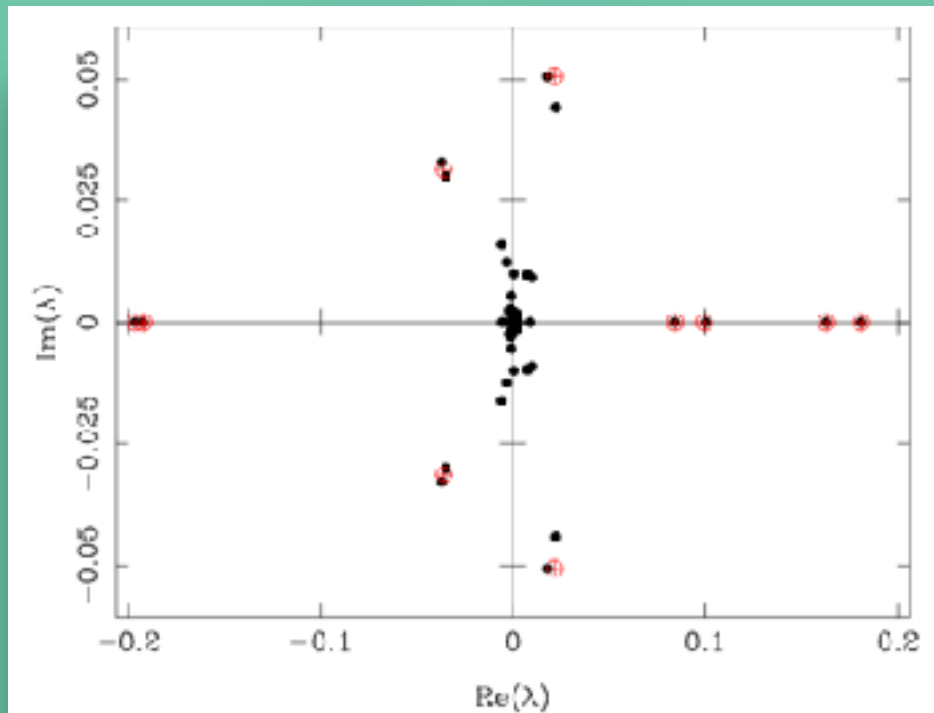
matriz chica (dependiendo de $1 - |\alpha_1|$). Necesita evolución eficiente

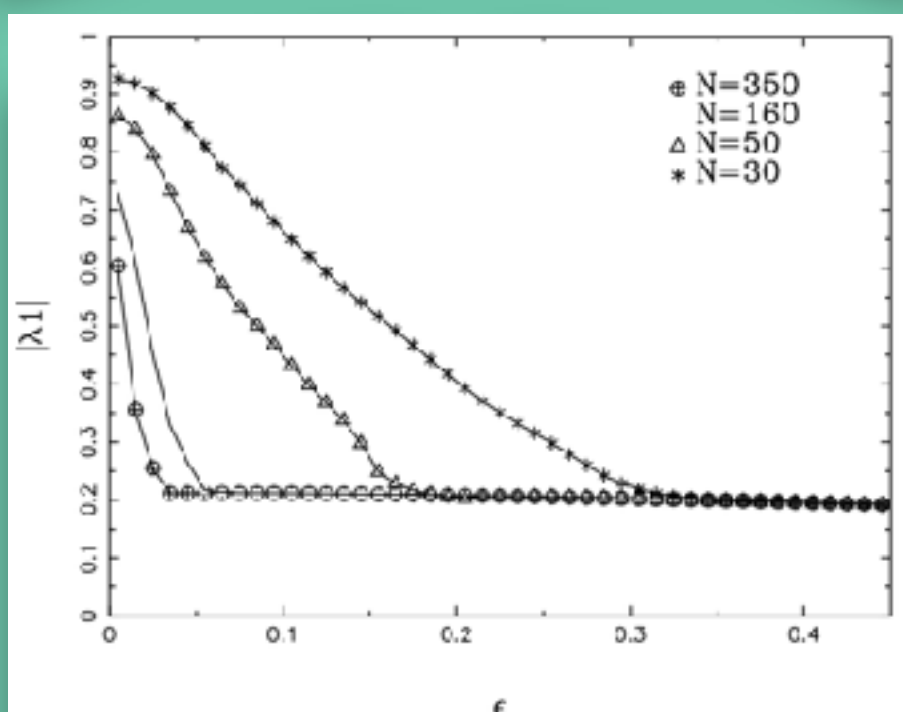


$1 > |\alpha_1| > |\alpha_2| > \ldots$

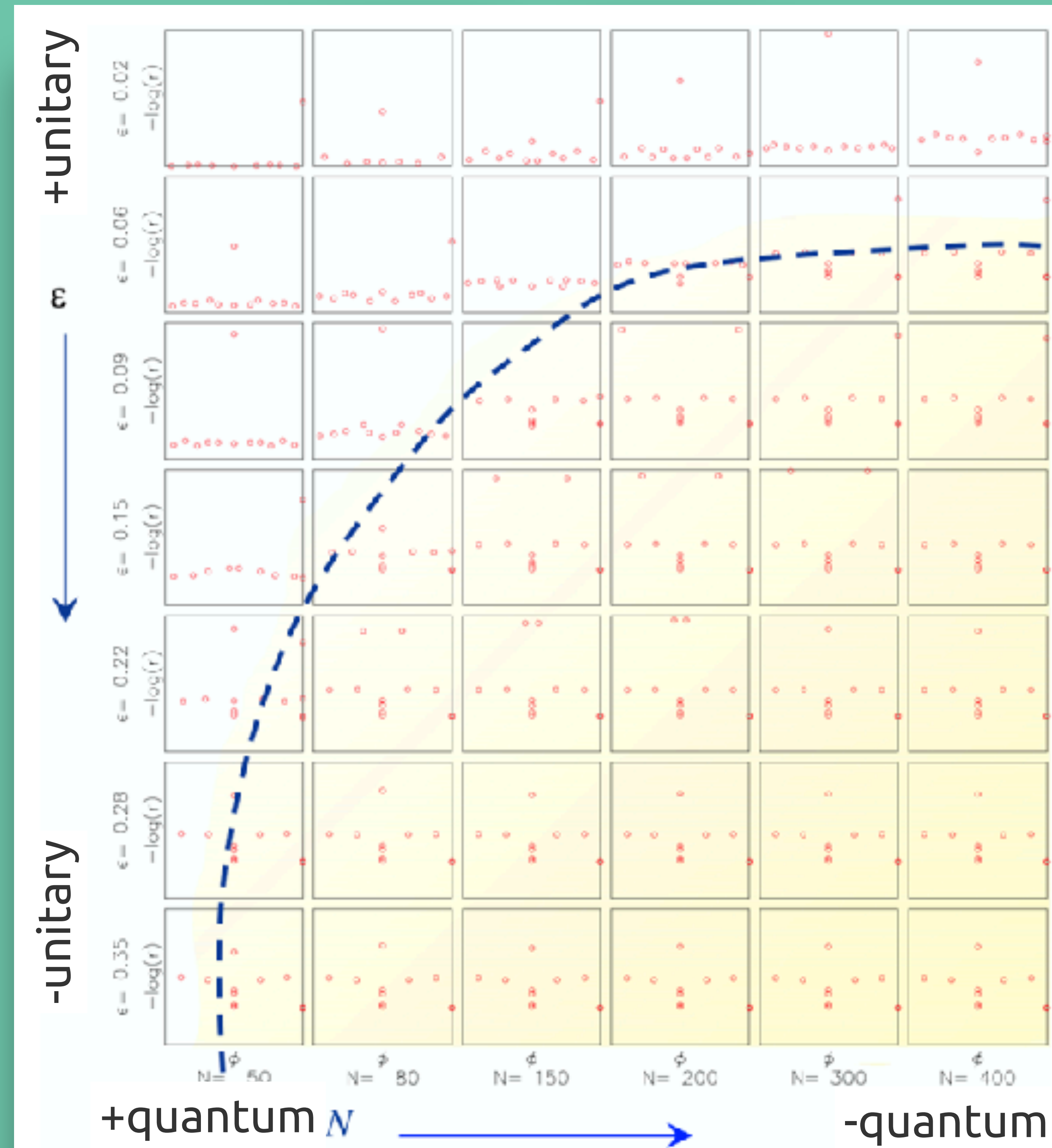
truncar

matriz más grande (~O(N)), más estable





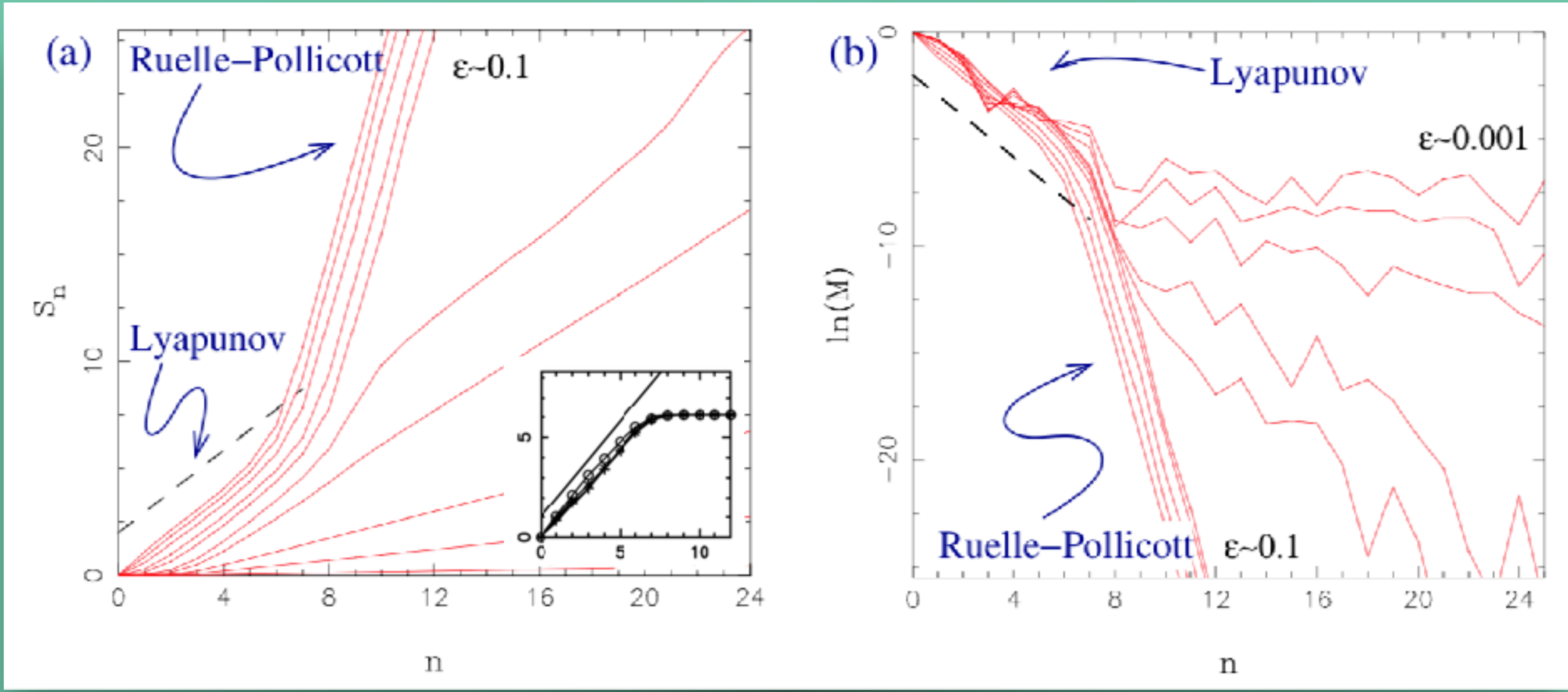
IGM, M Saraceno, and M E Spina. PRL (2003)



IGM and M Saraceno. PRE (2004)

$-\ln[\mathrm{tr}(\rho^2)]$ pureza

eco



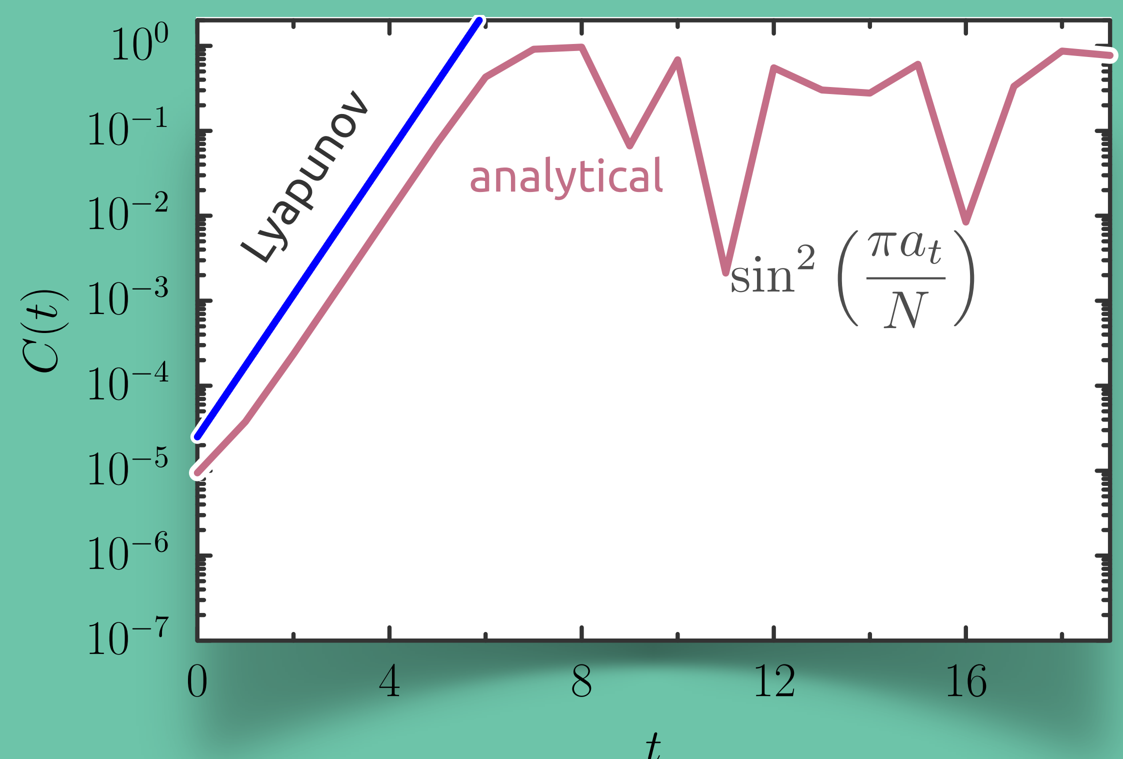
IGM and M Saraceno. PRE (2004)

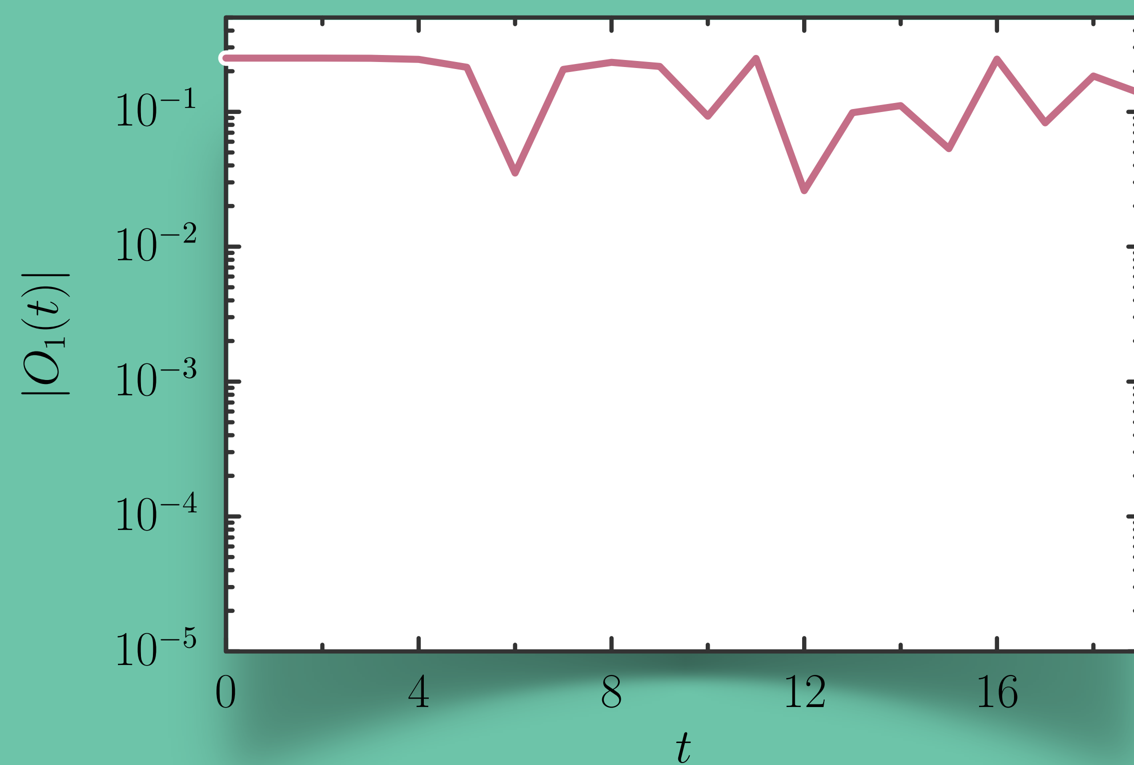
OTOC resultados numéricos

$$\hat{X}_t = (\hat{U}^\dagger_{cat})^t \hat{X} \hat{U}^t_{cat}$$

$$\mathcal{C}(t) = -\frac{1}{N}\mathrm{Tr}\left(\left[\hat{X}_t, \hat{P}\right]^2\right)$$

$$O_1(t) = \frac{1}{N}\mathrm{Tr}\left(A_t B A_t B\right)$$





$k = 0$ no-scrambling

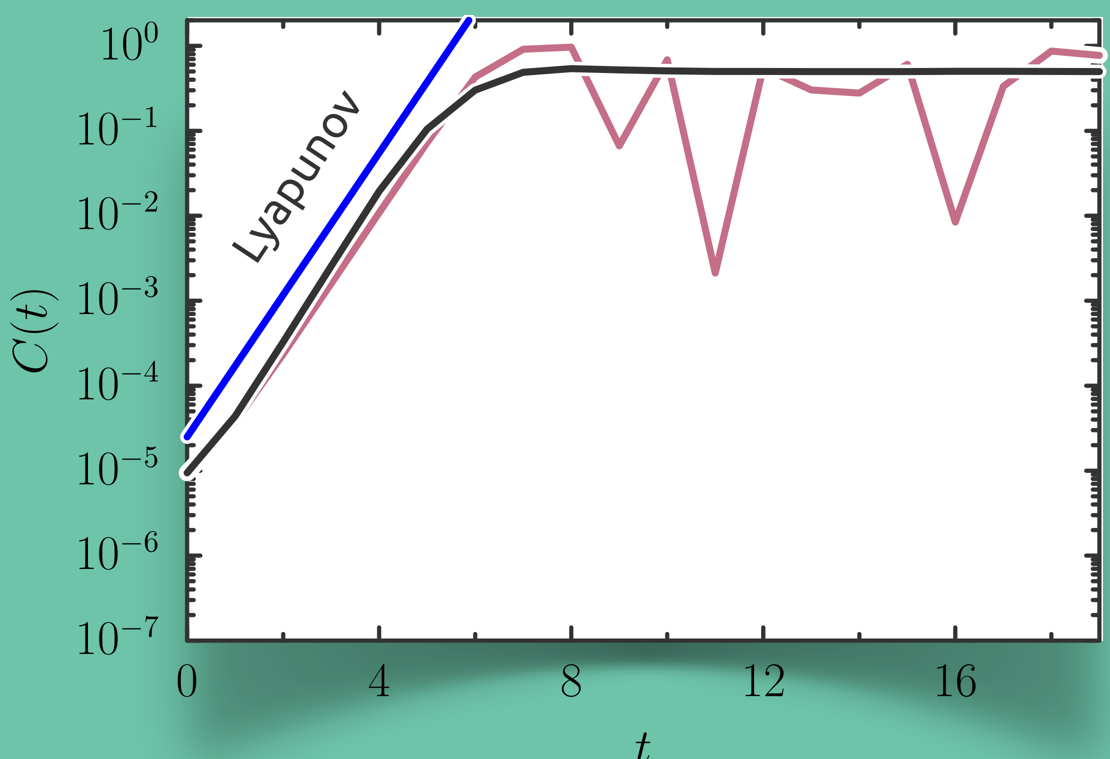
$$\hat{T}_{M^t \xi} = \hat{U}_M^{\dagger t} \hat{T}_\xi \hat{U}_M^t .$$

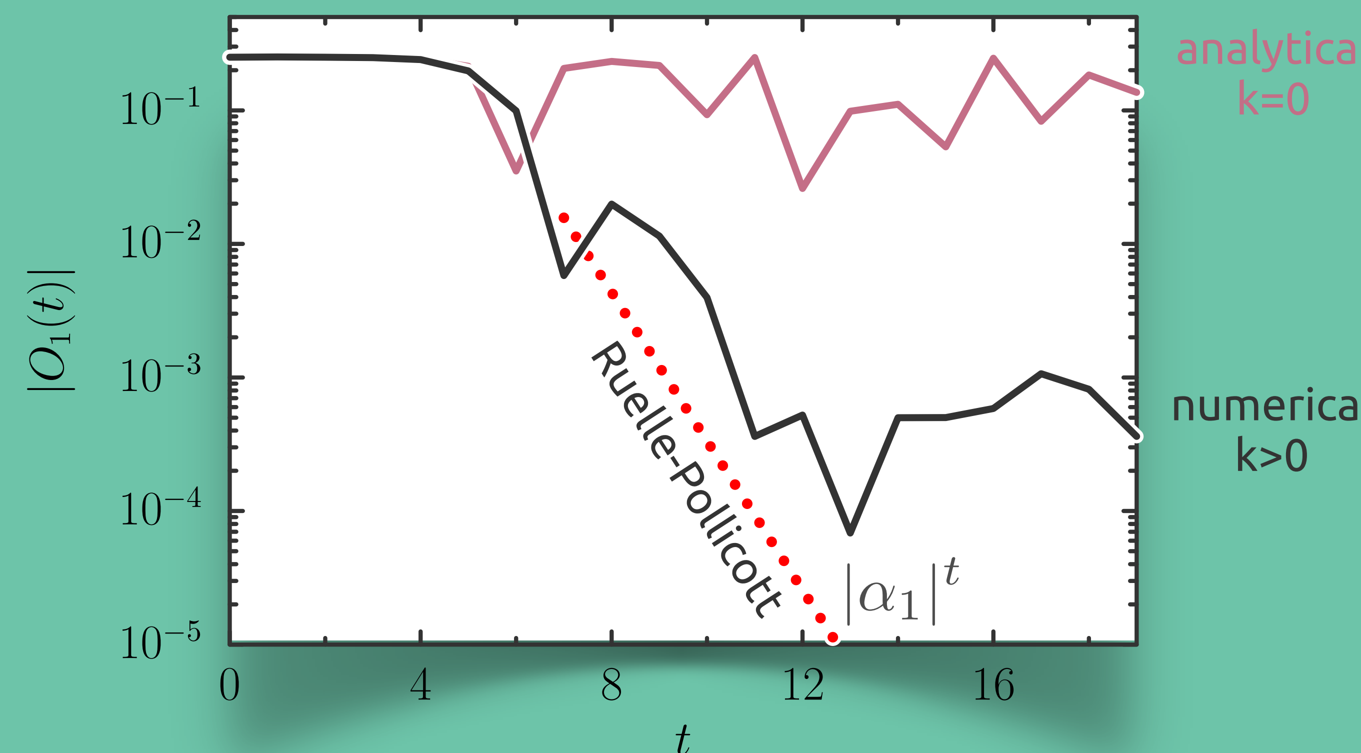
$$\hat{U}_{pcat} = e^{-i2\pi\left(\frac{p^2}{2N} - kN\cos(2\pi p/N)\right)} e^{-i2\pi\left(\frac{q^2}{2N} + kN\cos(2\pi q/N)\right)}$$

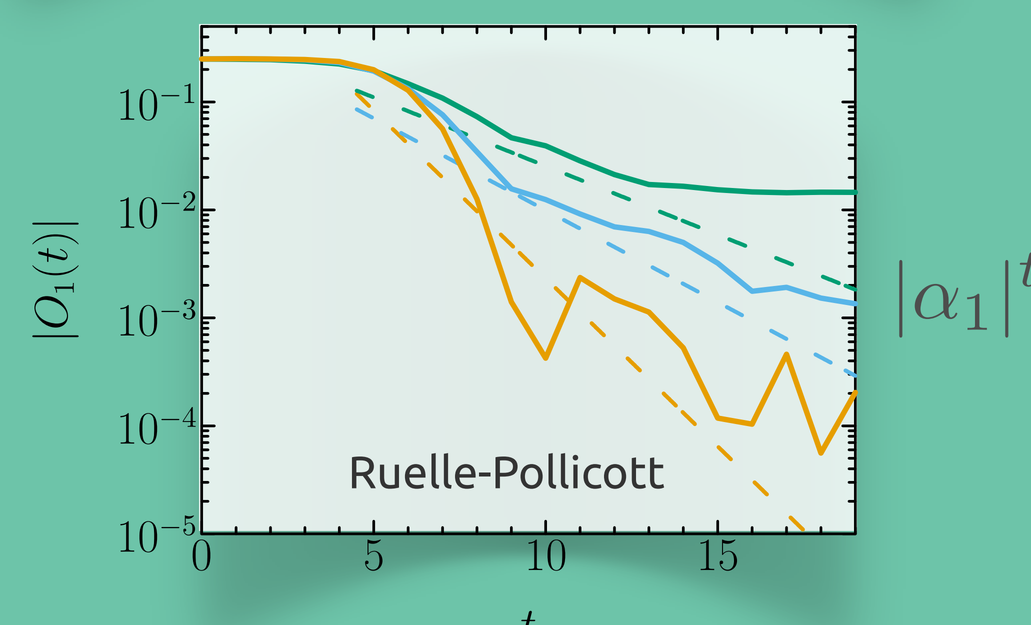
$$\hat{X}_t = (\hat{U}^\dagger_{pcat})^t \hat{X} \hat{U}^t_{pcat}$$

$$\mathcal{C}(t) = -\frac{1}{N}\mathrm{Tr}\left(\left[\hat{X}_t, \hat{P}\right]^2\right)$$

$$O_1(t) = \frac{1}{N}\mathrm{Tr}\left(A_t B A_t B\right)$$



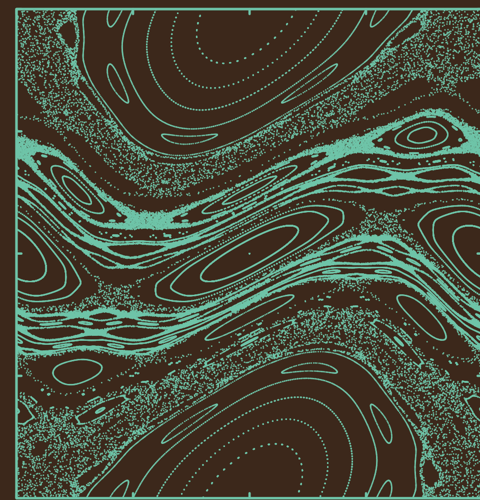




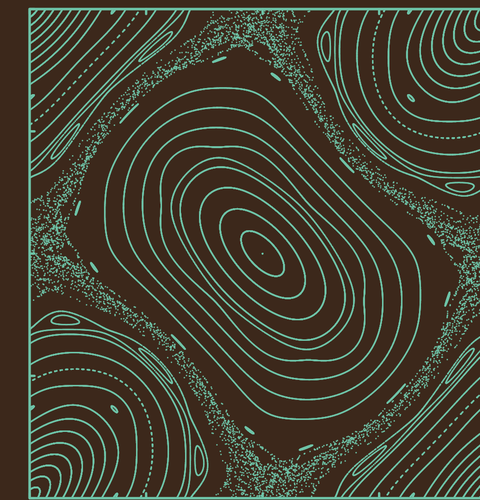
Ruelle está en $O_1(t) = \frac{1}{N}\mathrm{Tr}\left[\hat{A}_t\hat{B}\hat{A}_t\hat{B}\right]$

a.k.a OTOC (a veces)

standard

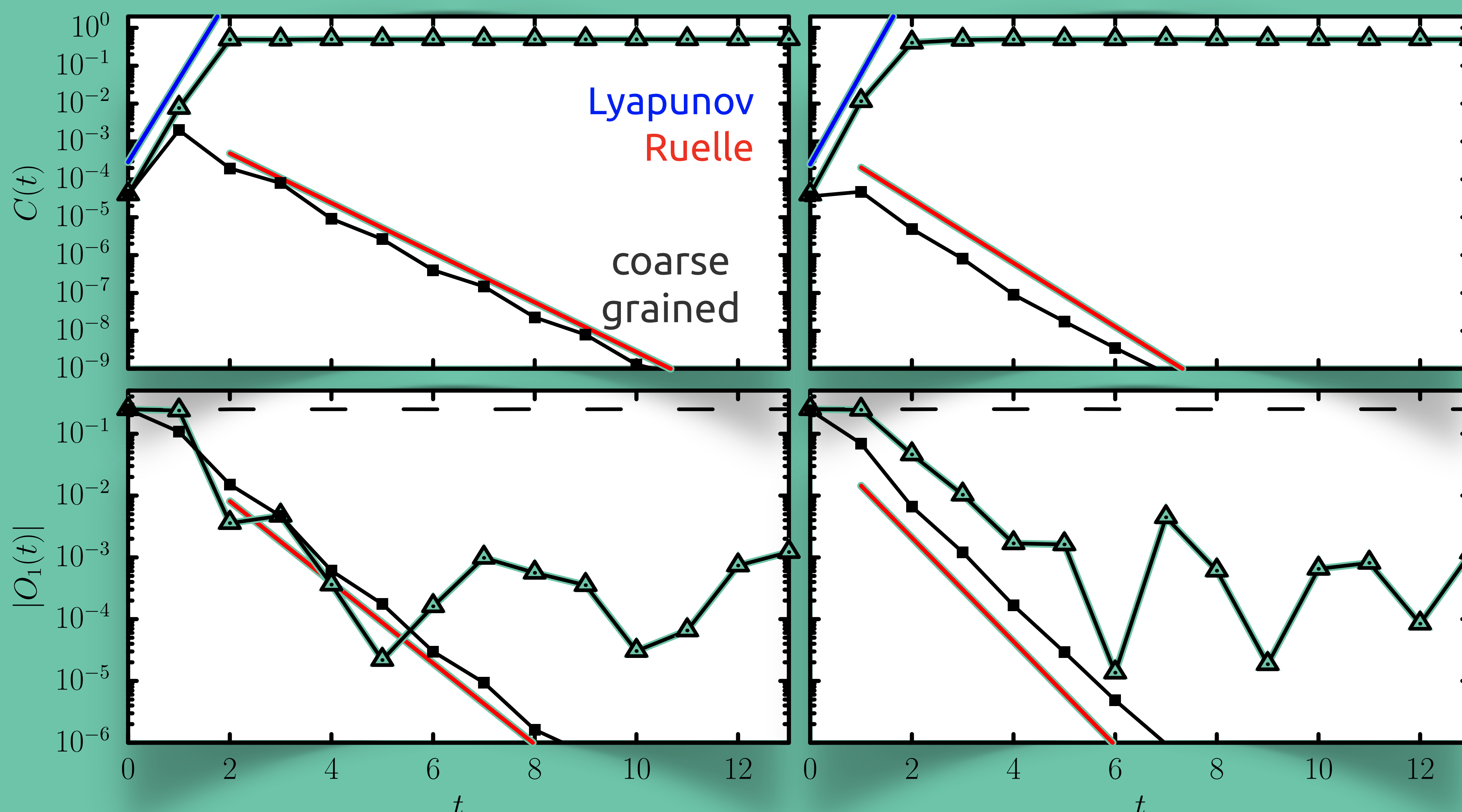


Harper



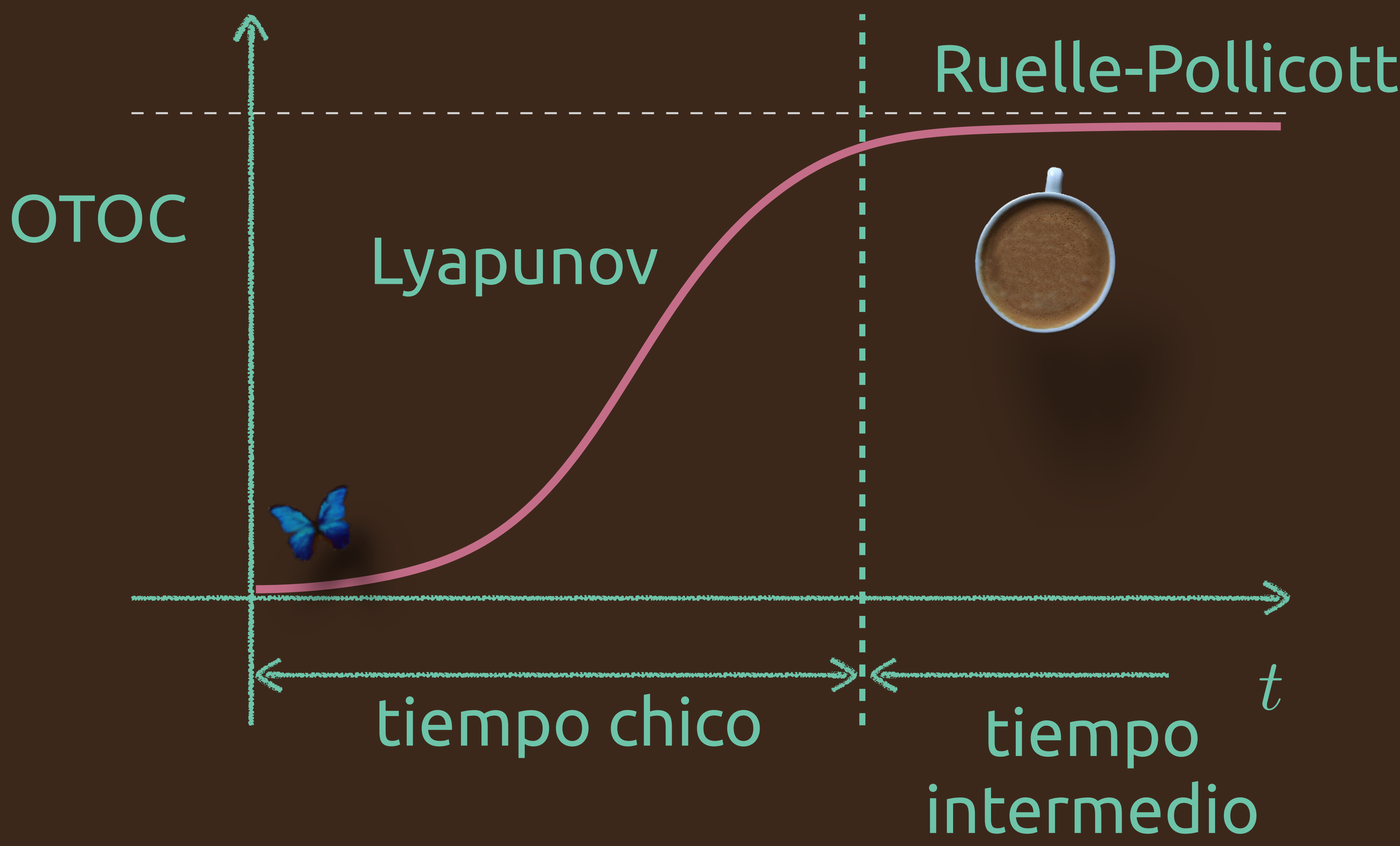
$$\hat{U}_K^{(\text{Standard})} = e^{-i\pi\frac{p^2}{N}} e^{-i2\pi K N \cos(2\pi q/N)}$$

$$\hat{U}_K^{(\text{Harper})} = e^{i2\pi N K \cos(2\pi p/N)} e^{i2\pi N K \cos(2\pi q/N)}$$

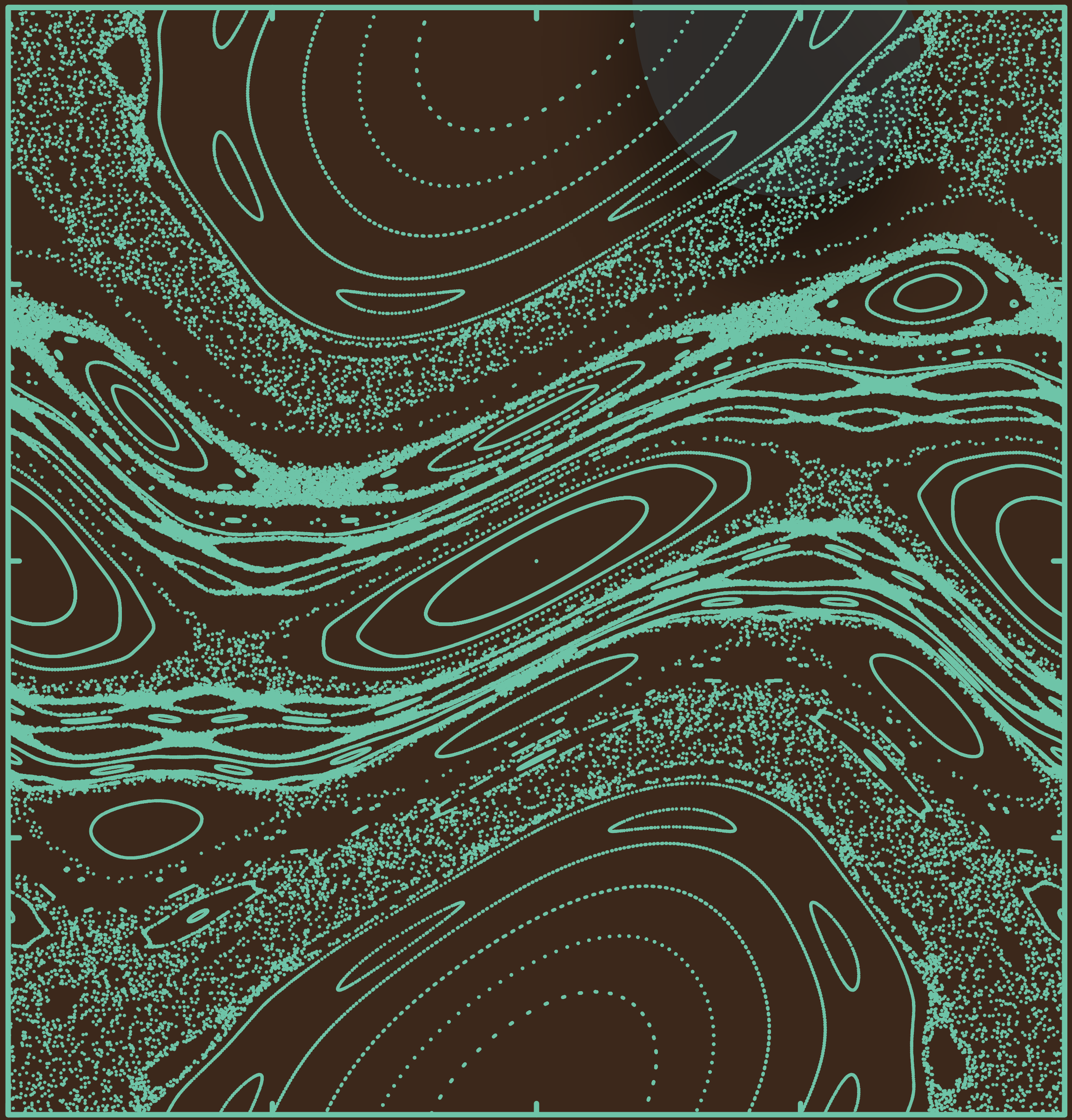


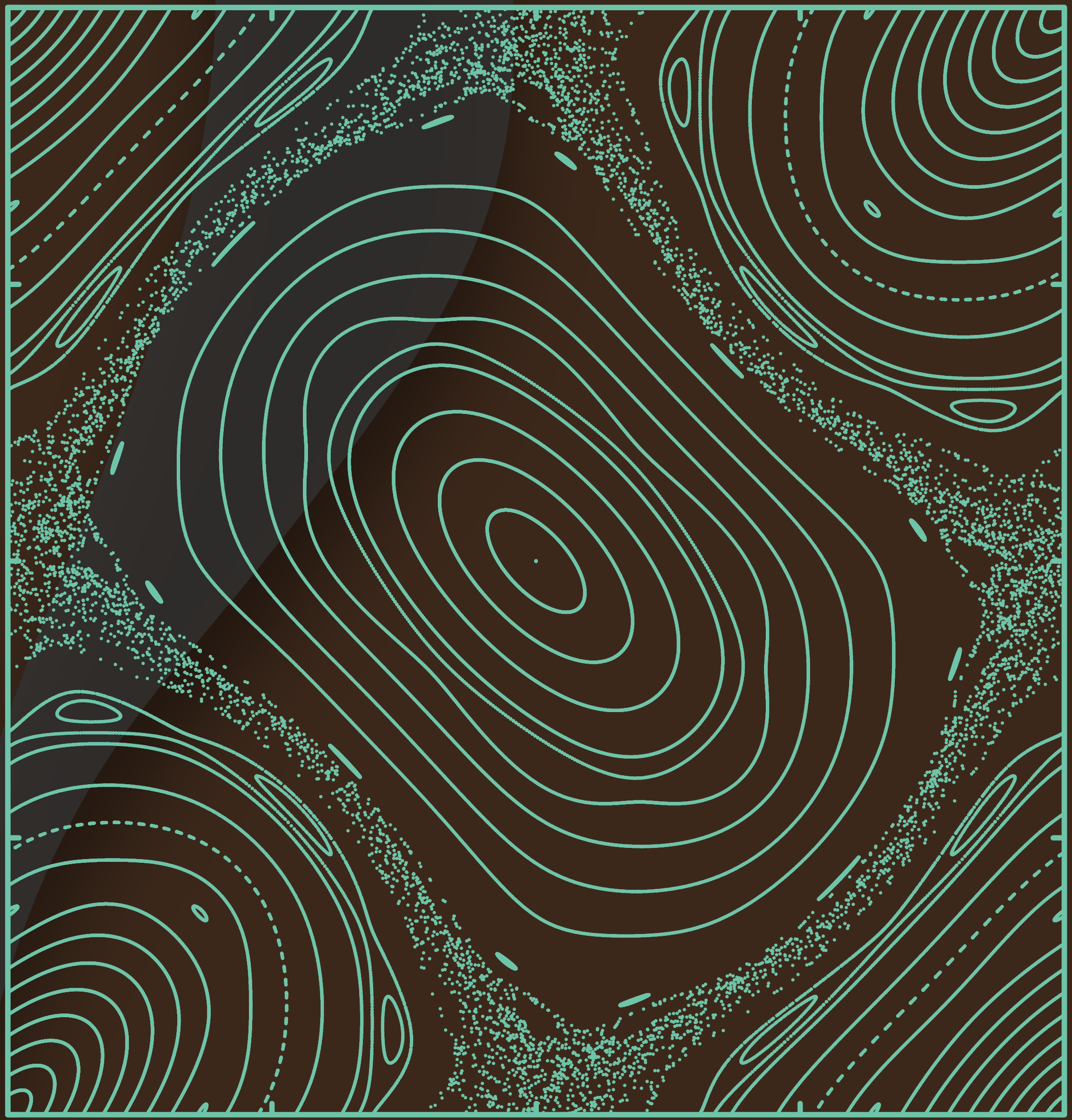
Caos

OTOC ~~debería tener~~ tiene los dos (Polchinsky 2015)



mezclado

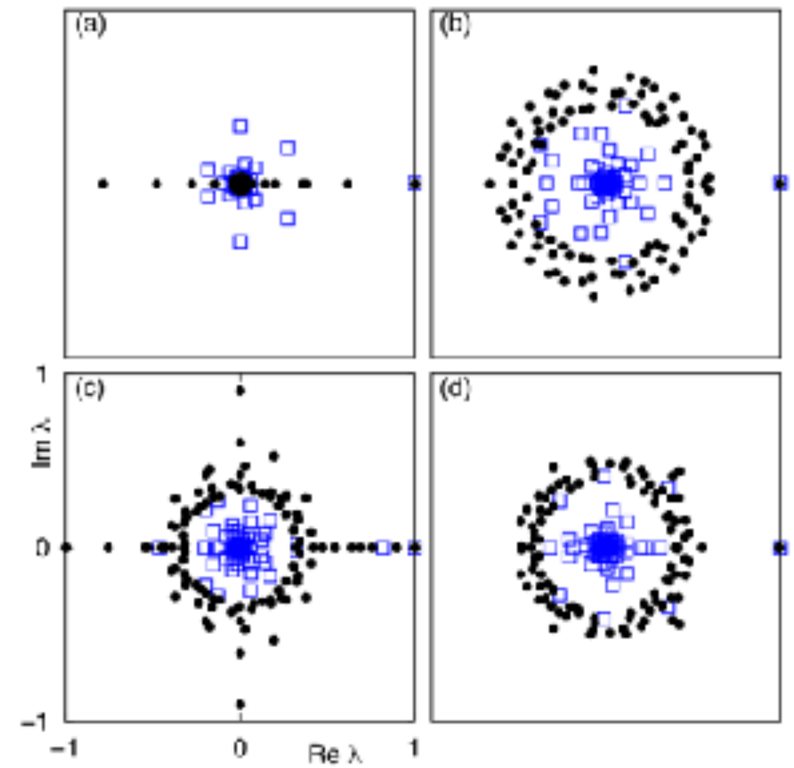




Quantum Lyapunov exponent in dissipative systems

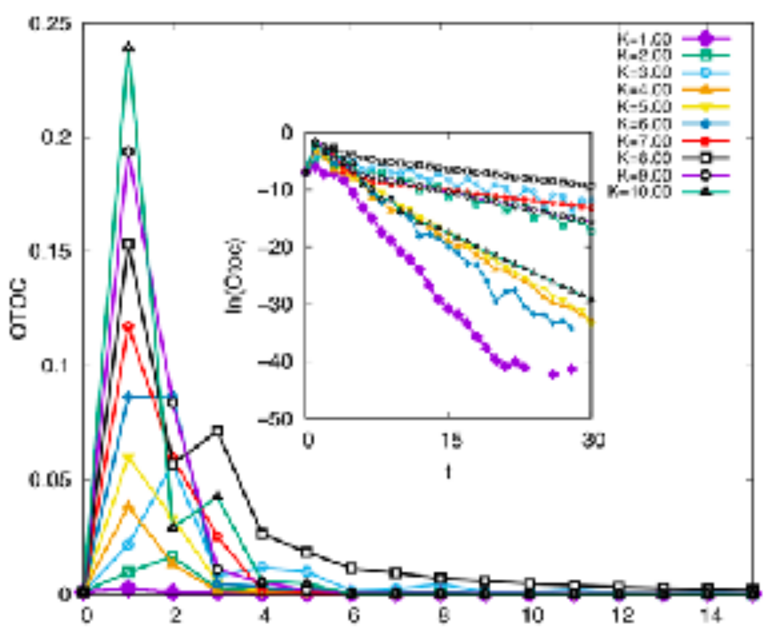
Pablo D. Bergamasco, Gabriel G. Carlo, and Alejandro M. F. Rivas
Phys. Rev. E **108**, 024208 – Published 8 August 2023

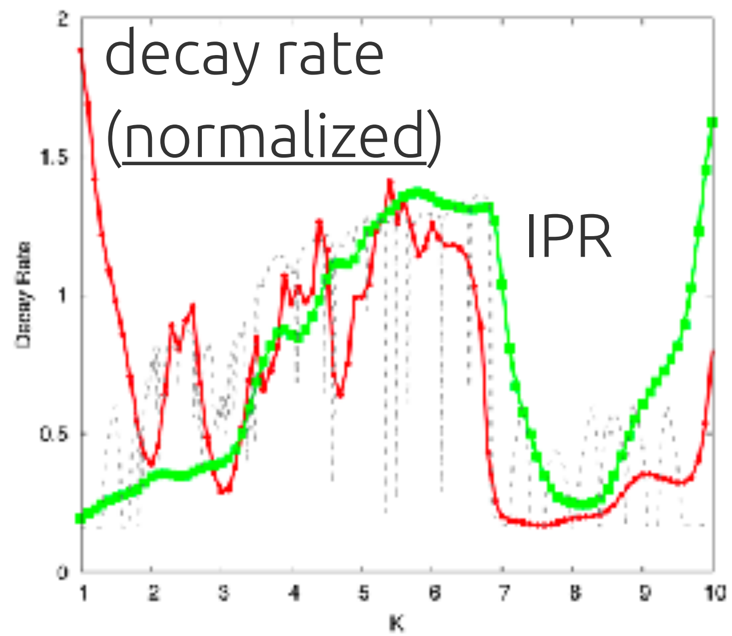
Ulam+ruido



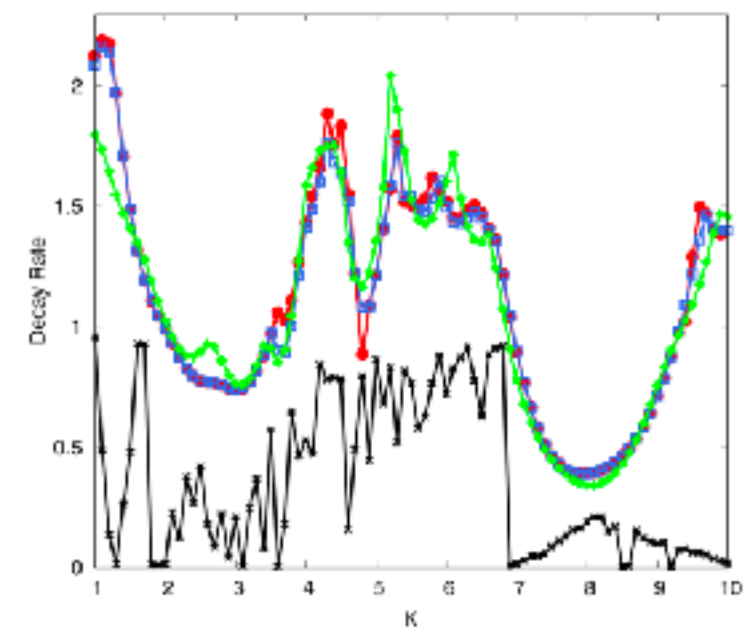
particula en 1d , potencial pateado asimétrico

entorno modelado con operadores de Lindblad tipo escalera





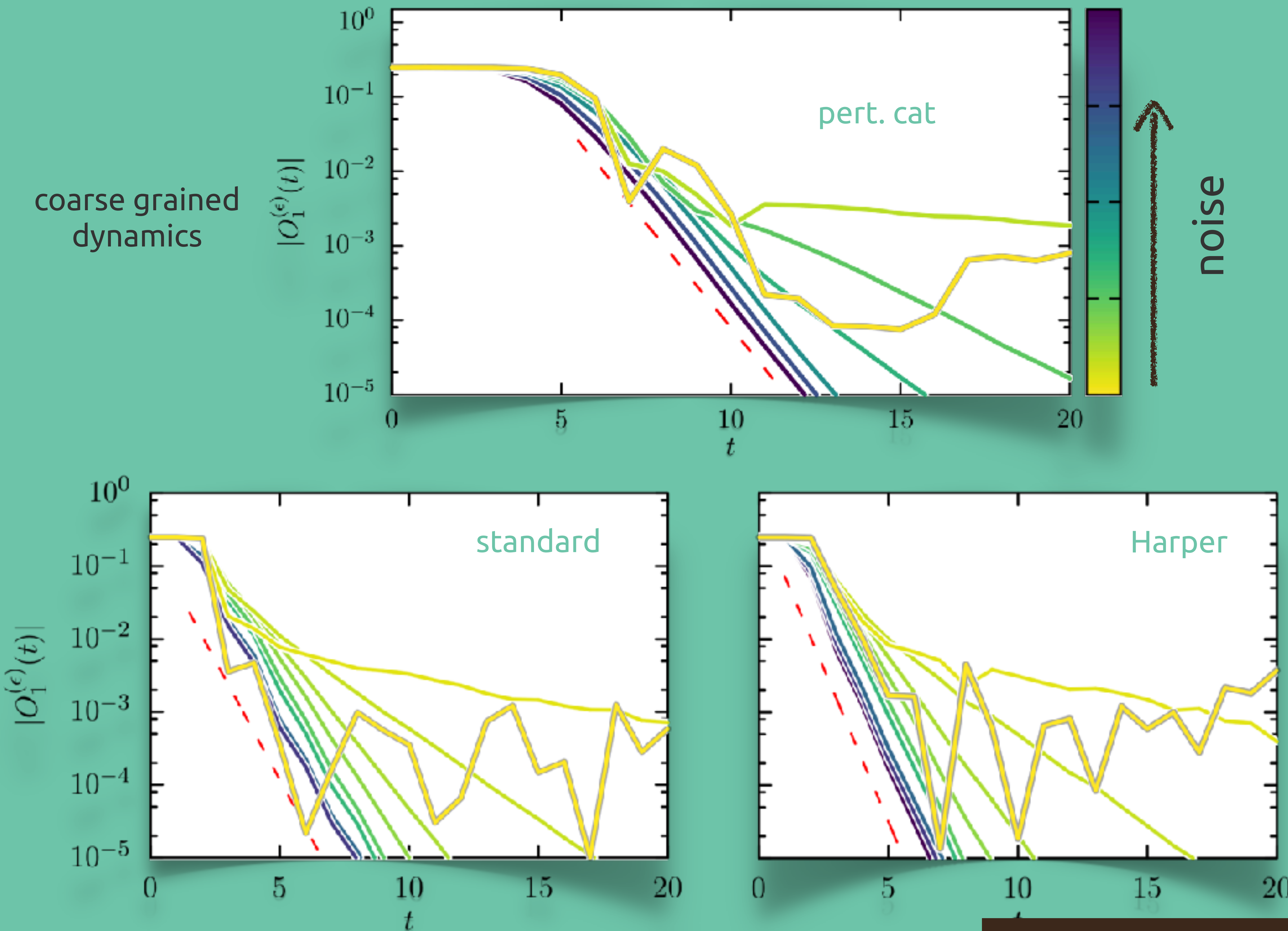
dashed =Lyapunov



decay rate=largest EV (w noise)

Nonnenmacher (2003) & our PRL (2018)

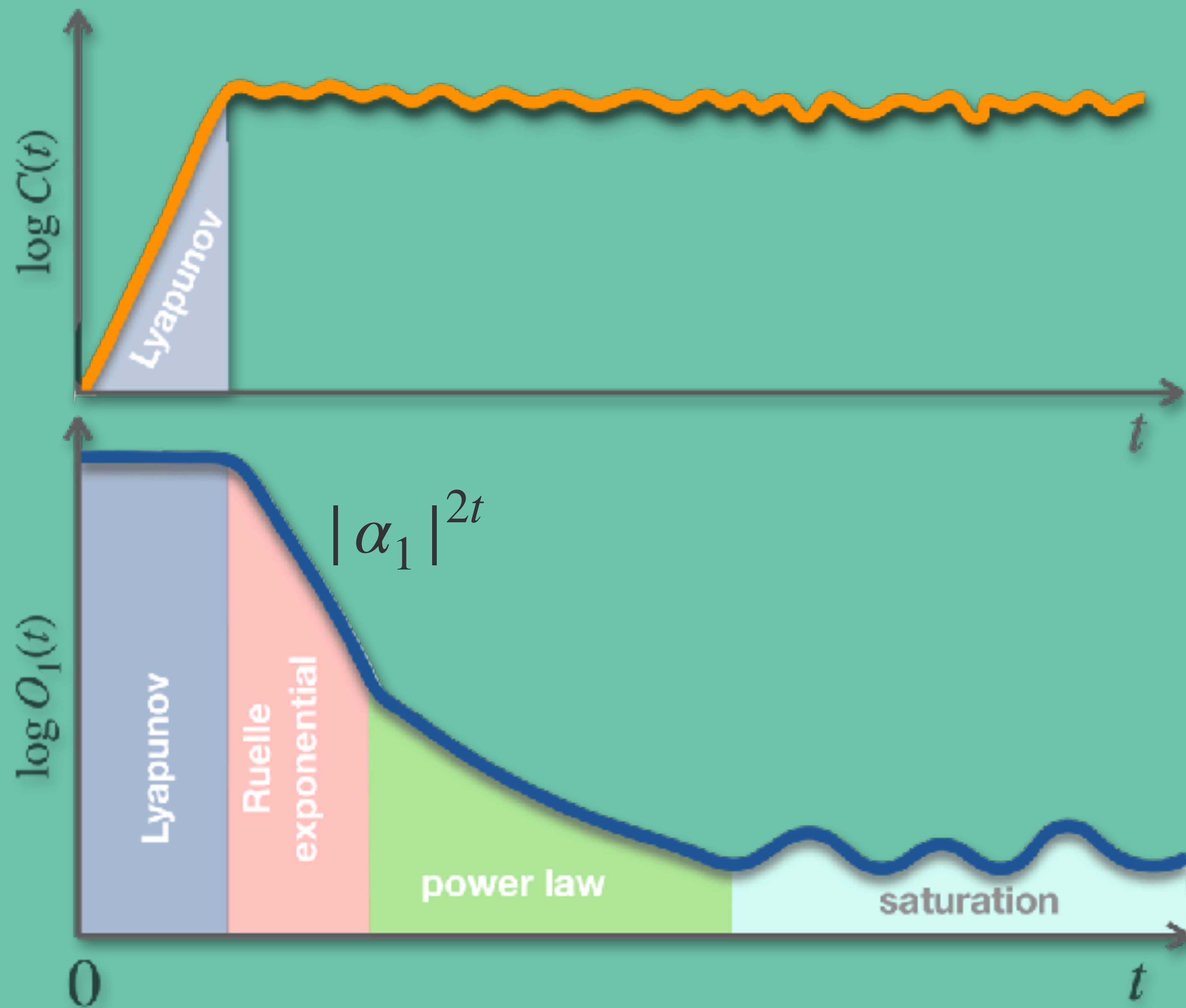
coarse grained dynamics

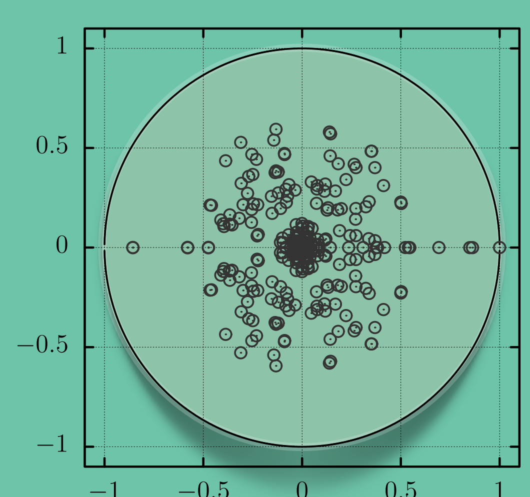


nuestro PRL (2018)

no coarse graining

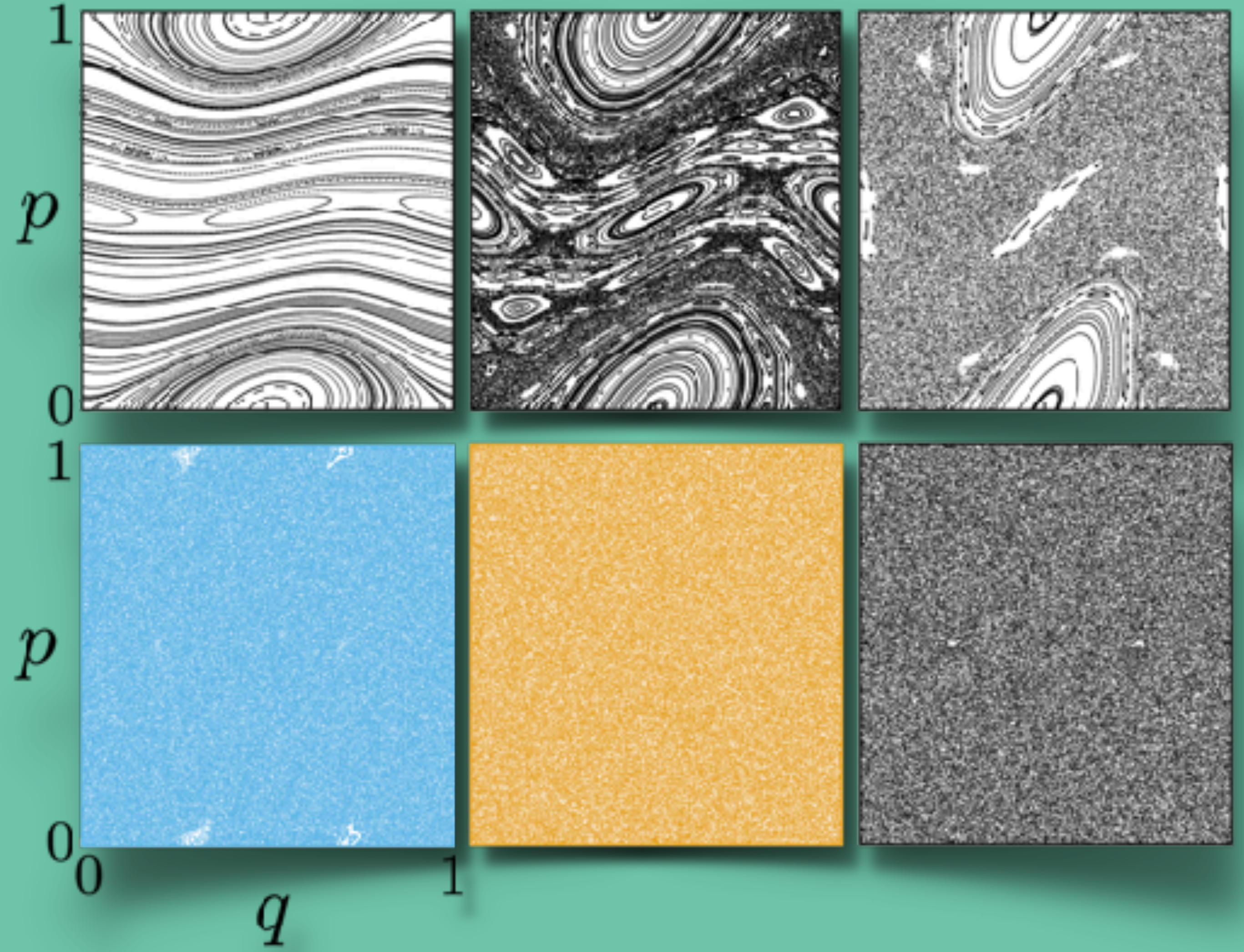
no decoherence



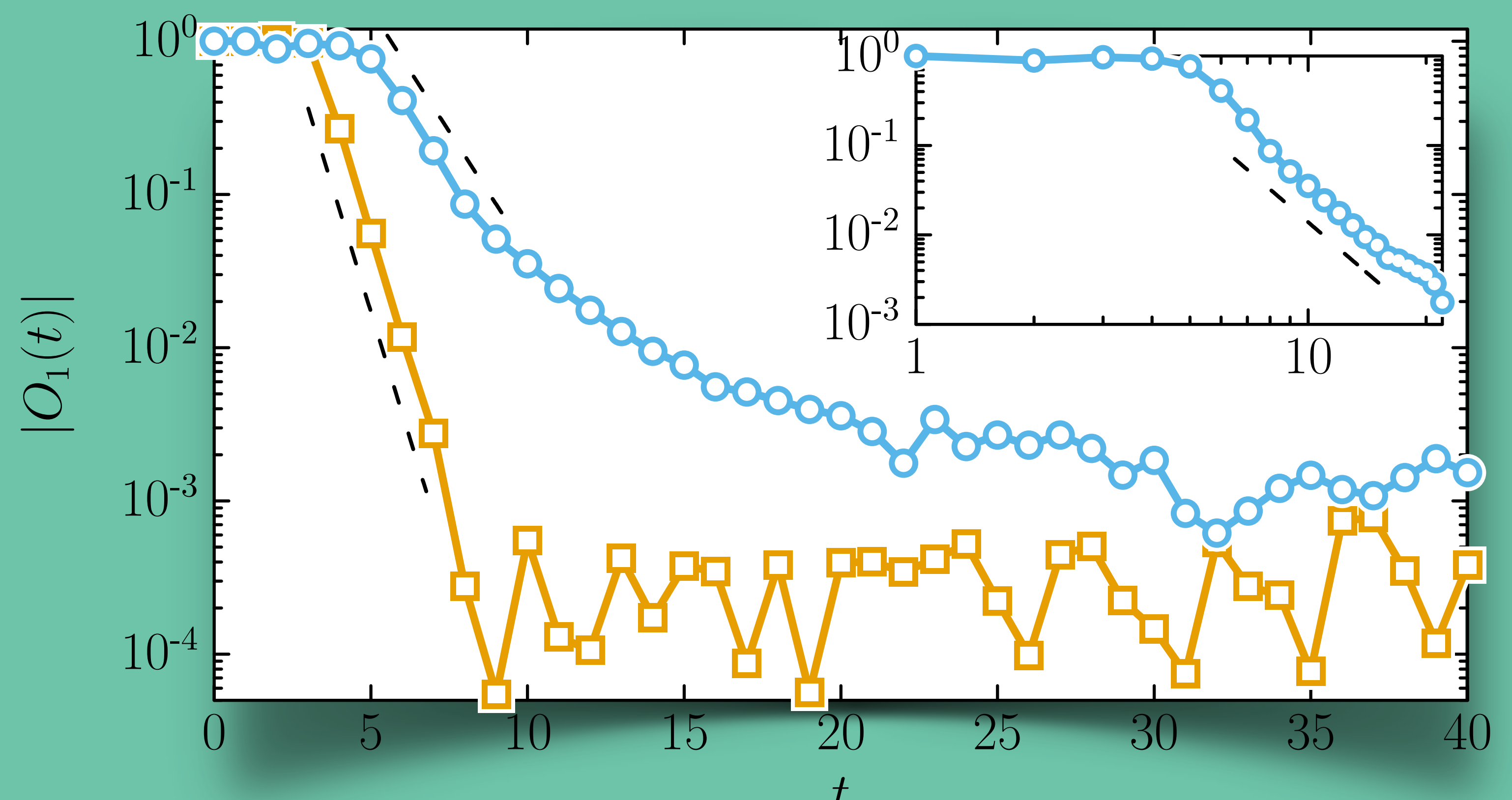


$1 > |\alpha_1| > |\alpha_2| > \ldots$

point spectrum of the Perron-Frobenius operator
(in some special basis)



p
p
1
0
1
0
0
1
q



$|O_1(t)|$
t
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0
5
10
15
20
25
30
35
40
1
10

diagonalizar Perron-Frobenius

coarse grained PF

Blum and Agam, PRE 62, 1977 (2000)

$$(q', p'|\mathcal{L}|q, p) = \delta\left\{p' - \left[p + \frac{K}{2\pi}\sin(2\pi q)\right]\right\} \times \delta\left(q' - [q + p']\right)$$

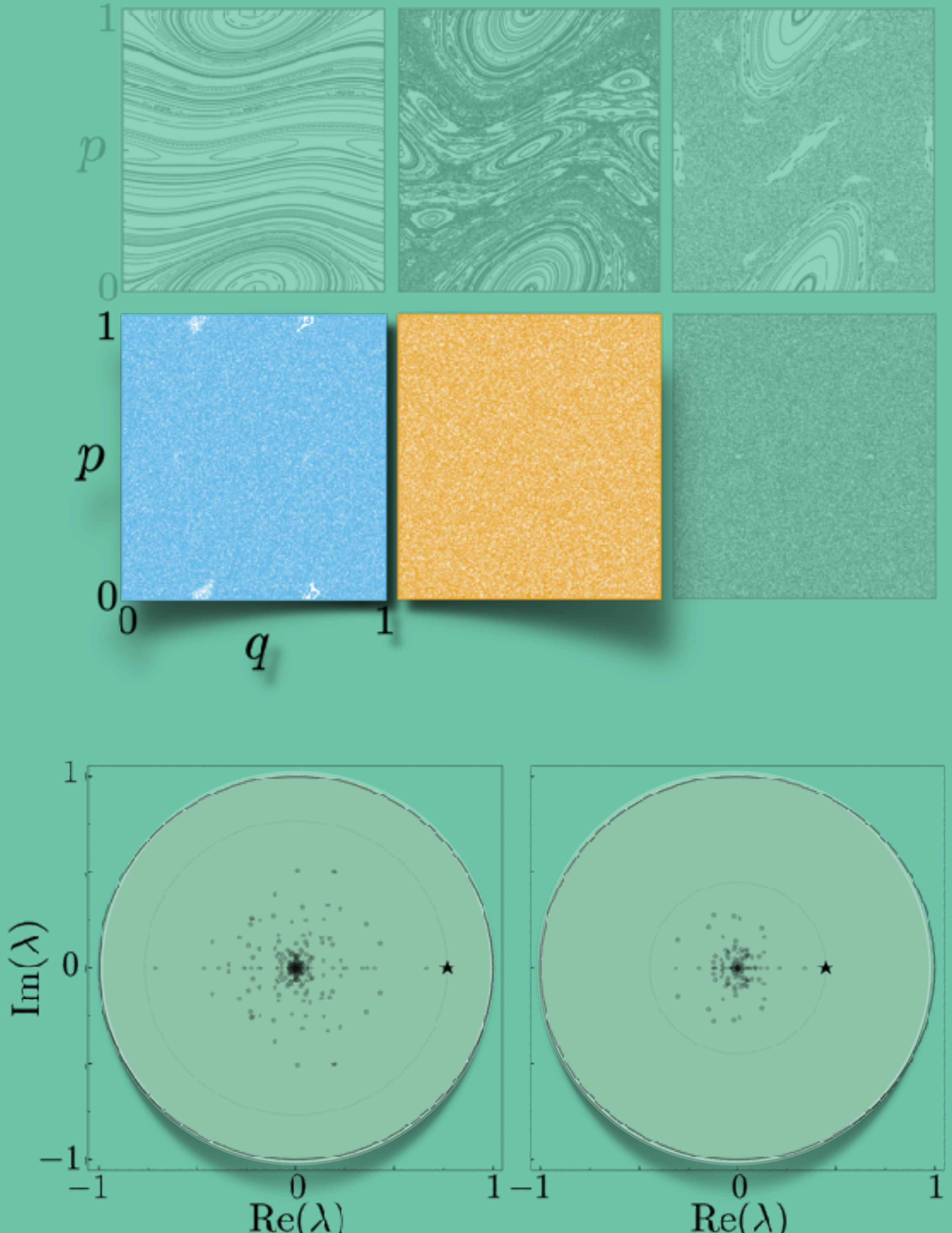
$$\delta(x) \to \sum_j \frac{1}{\pi s}\exp\left[-(x-j)^2/s\right]$$

Fourier transformed phase space + σ variance noise

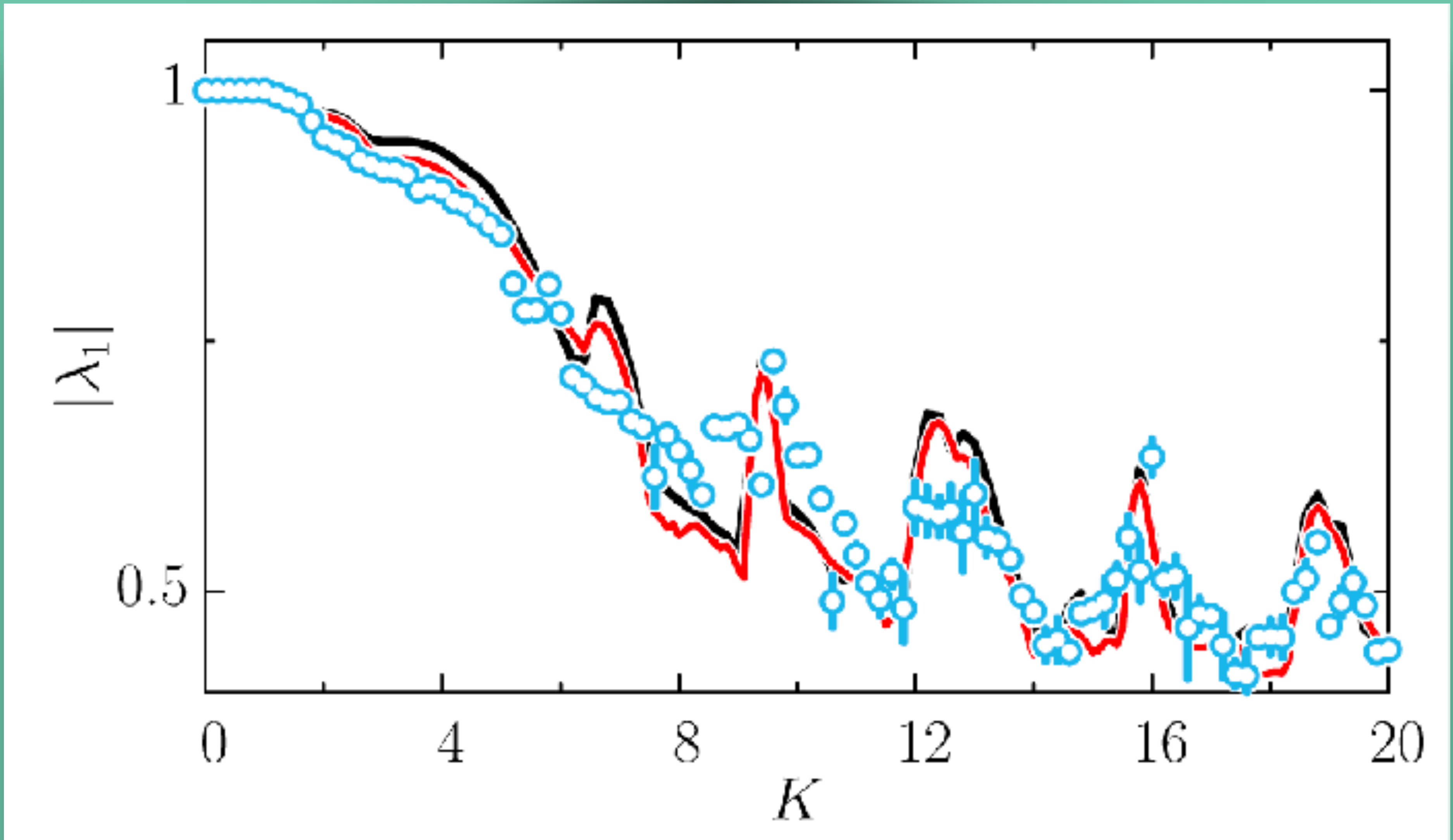
Khodas and Fishman, PRL. 84, 2837 (2000)

$$(q, p \mid k, m) = \exp(imq)\exp(ikp)/(2\pi)$$

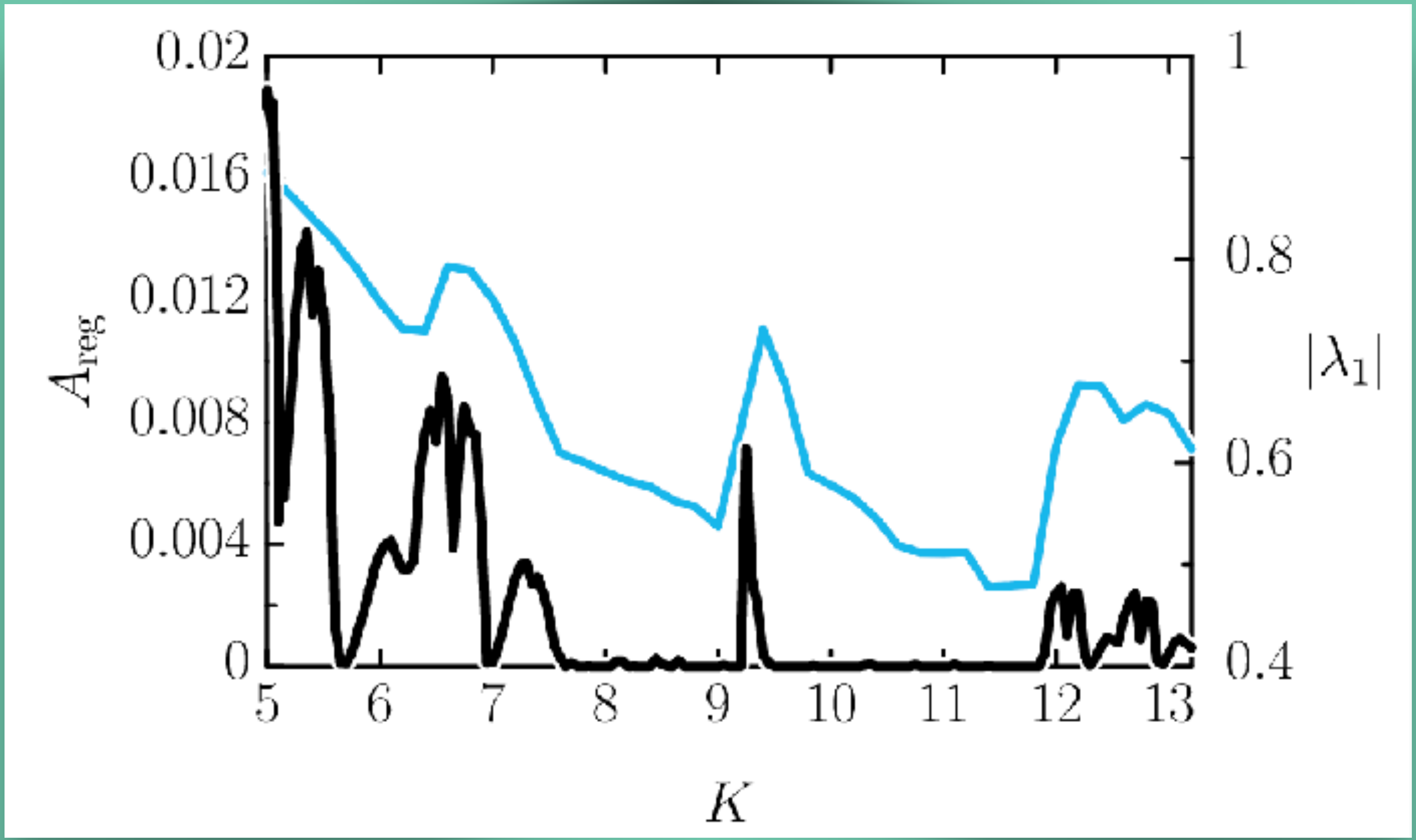
$$\left(k, m \left| \mathcal{L}^{(\sigma)} \right| k', m'\right) = J_{m-m'}(k'K)\exp\left(-\frac{\sigma^2}{2}m^2\right)\delta_{k-k',m},$$



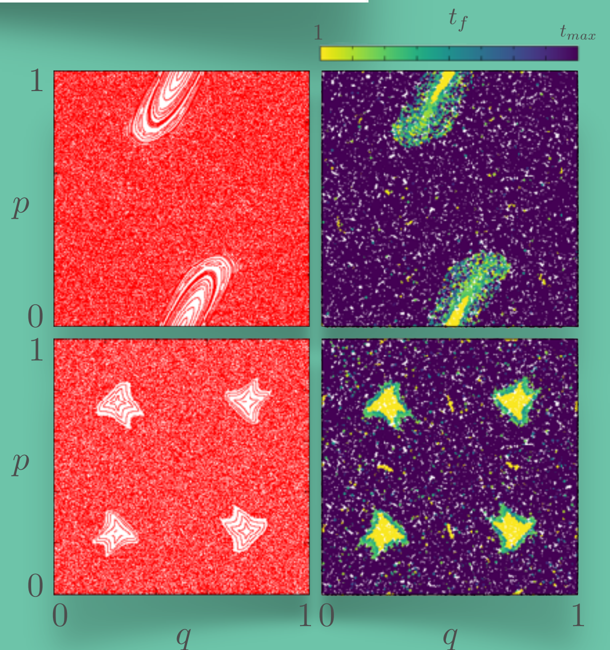
1
p
0
1
p
0
0
q
1
1
0
−1
Im(λ)
−1
0
1
Re(λ)
−1
0
1
Re(λ)



1
0.5
$|\lambda_1|$
0
4
8
12
16
20
K



0.02
0.016
0.012
0.008
0.004
0
A_{reg}
1
0.8
0.6
0.4
$|\lambda_1|$
5
6
7
8
9
10
11
12
13
K



t_f
1
t_{max}
p
q
0
1

PHYSICAL REVIEW E **107**, 064207 (2023)

Classical approach to equilibrium of out-of-time ordered correlators in mixed systems

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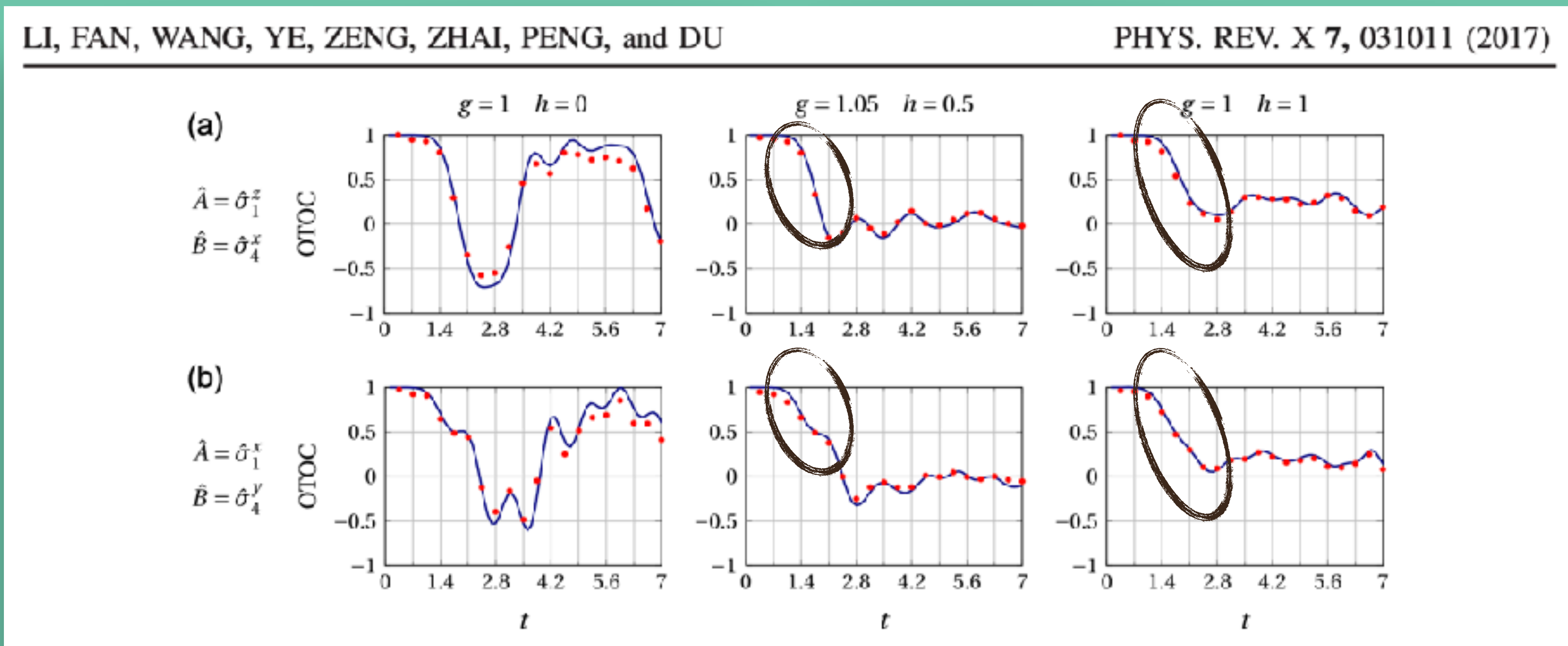
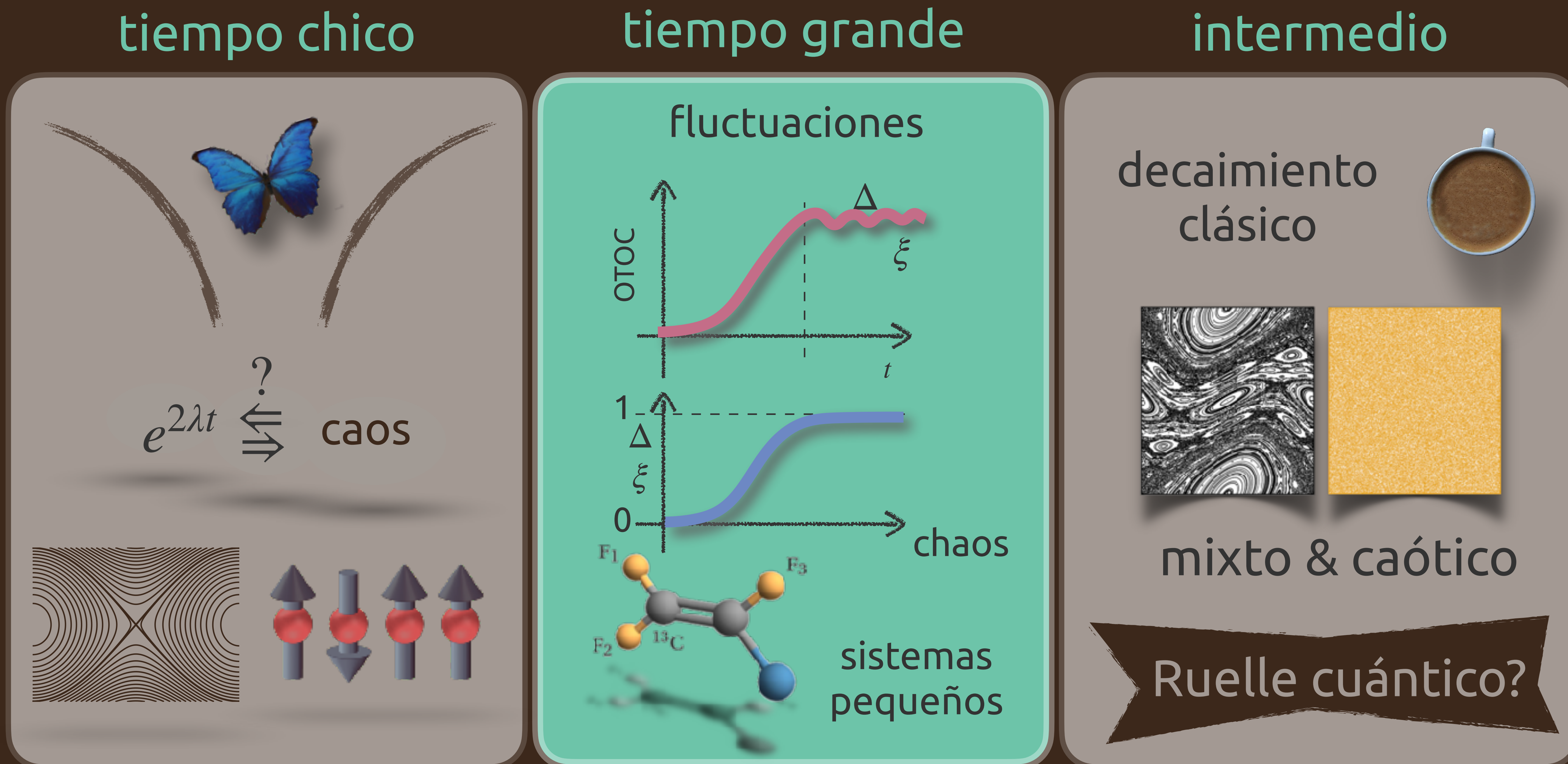
outlook

quantum Ruelle?





tiempo chico
tiempo grande
intermedio
$e^{2\lambda t}$? caos
fluctuaciones
OTOC
Δ
ξ
t
1
0
chaos
F1
F2
F3
13C
sistemas pequeños
decaimiento clásico
mixto & caótico
Ruelle cuántico?

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PRE **107**, 064207 (2023). `arXiv:2303.08047`

gracias

funding

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Additional

Stuff

$$\hat{A}_{t+1} = \mathbf{S}_\epsilon(\hat{A}_t) \equiv \mathbf{D}_\epsilon(\hat{U}^\dagger \hat{A}\hat{U})$$

Diagonalize this $N^2 \times N^2$ superoperator ideally for large N

$$\mathbf{D}_e(\hat{O}) \stackrel{\text{def}}{=} \sum_\xi c_\epsilon \hat{T}_\xi^\dagger \hat{O} \hat{T}_\xi$$ is diagonal in the $\hat{T}_\xi$ basis

spectrum proportional to Fourier Transform of $c_\epsilon(\xi)$

OTOC for maps

$$\mathcal{C}(t) = -\frac{1}{N}\mathrm{Tr}\left[\hat{A}_t, \hat{B}\right]^2 = -\frac{2}{N}[O_1(t) - O_2(t)]$$

$$O_1(t) = \mathrm{Tr}\left[\hat{A}_t\hat{B}\hat{A}_t\hat{B}\right]$$

$$O_2(t) = \mathrm{Tr}\left[\hat{A}_t^2\hat{B}^2\right]$$

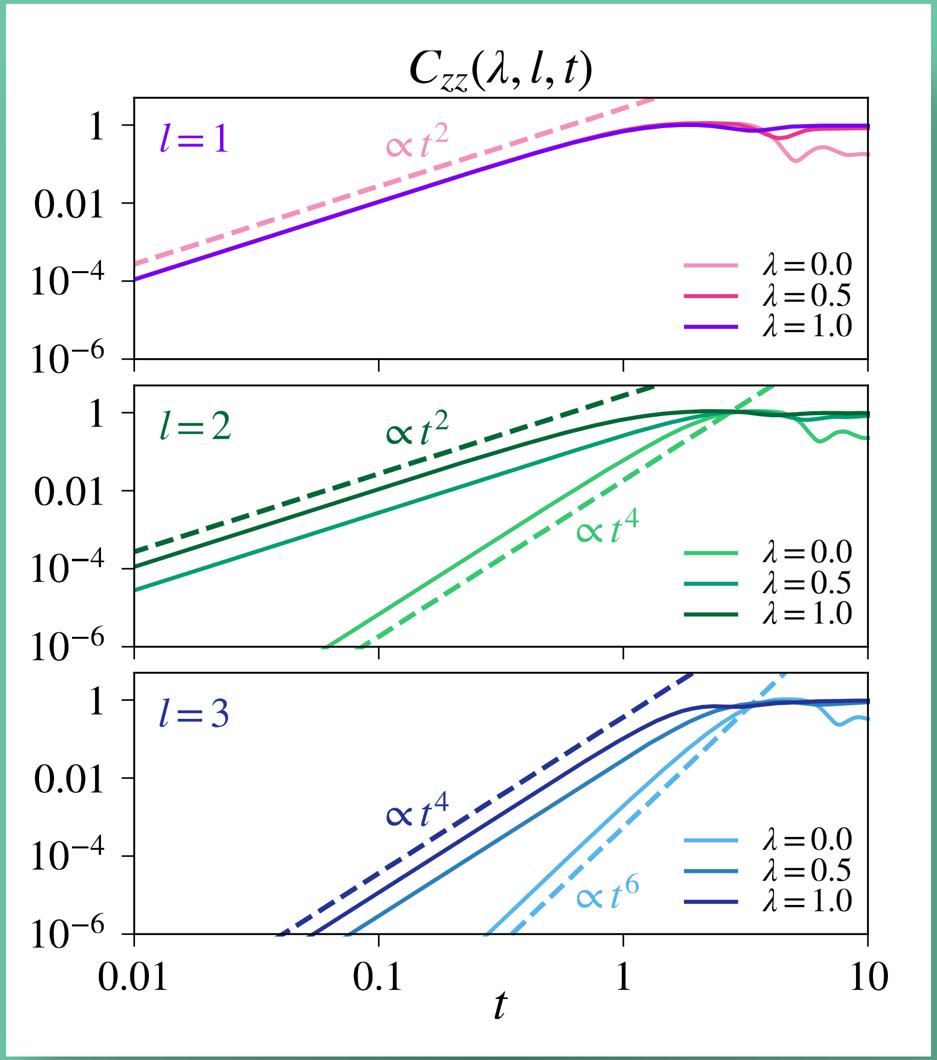
Perturbed XXZ model

e.g. L. F. Santos, F. Borgonovi, and F. M. Izrailev,
Phys. Rev. E 85, 036209 (2012)

$$\hat{H}(\lambda) = \hat{H}_0 + \lambda \hat{H}_1$$

$$\hat{H}_0 = \sum_{i=0}^{L-2} (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \mu \hat{S}_i^z \hat{S}_{i+1}^z),$$

$$\hat{H}_1 = \sum_{i=0}^{L-3} (\hat{S}_i^x \hat{S}_{i+2}^x + \hat{S}_i^y \hat{S}_{i+2}^y + \mu \hat{S}_i^z \hat{S}_{i+2}^z).$$



small t

$$C_{zz}(l,t) \approx \begin{cases} \frac{1}{2} \frac{t^{2l}}{(2l!)^2} & \text{if } l = 1 \; or \; \lambda = 0, l > 1 \\ \frac{1}{2} \frac{\lambda^{(2l-2)} t^{(2l-2)}}{((2l-2)!)^2} & \text{if } \lambda \neq 0, l \geq 2 \end{cases}$$

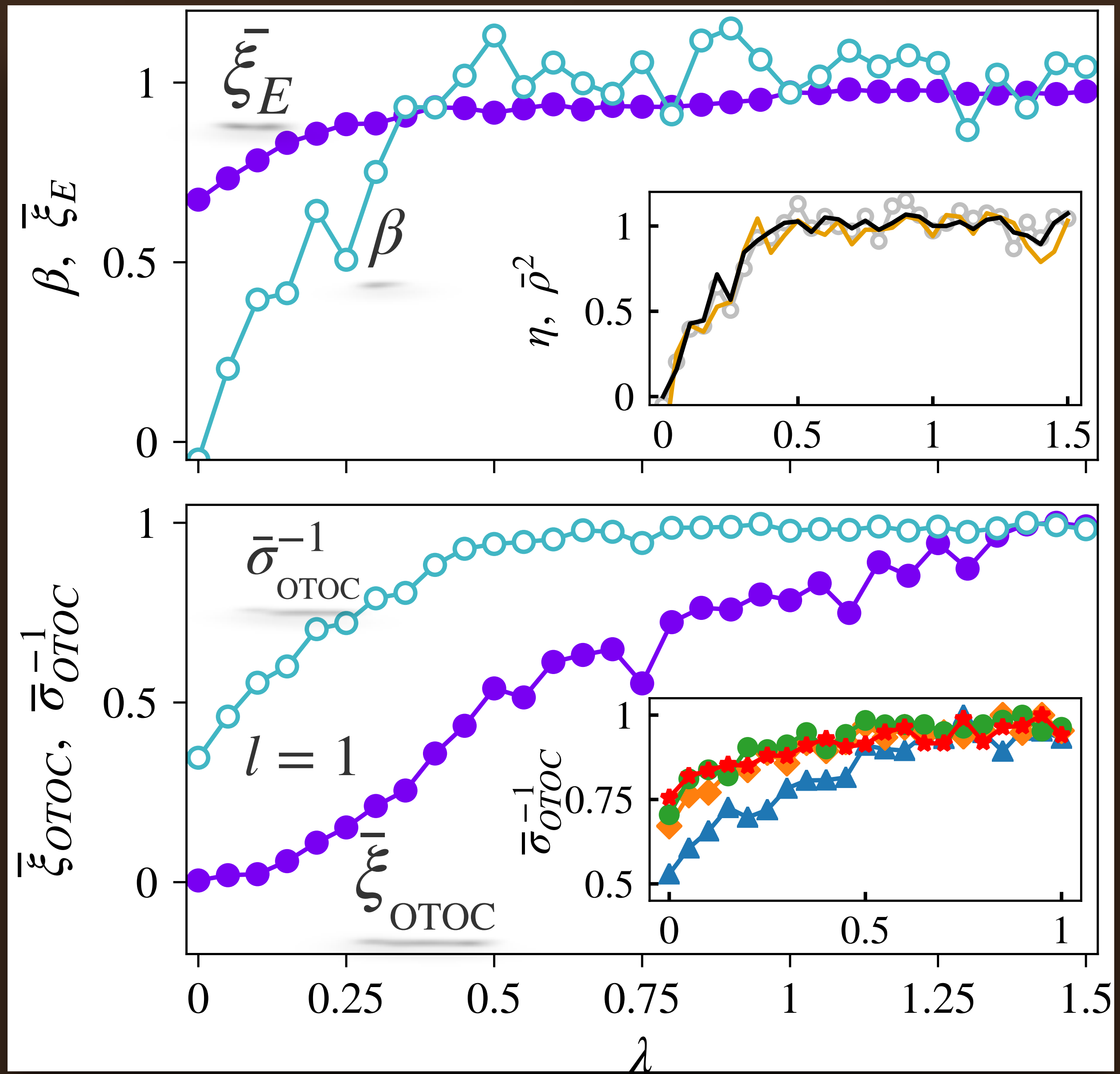
$\bar{\xi}_E$ spin site basis
10% center of the spectrum

Symmetry subspaces of dim

$$D_N \equiv \dim\left(\hat{\mathcal{S}}_N\right) = \left(\begin{array}{c} L \\ N \end{array}\right) = \frac{L!}{N!(L-N)!}$$

Parity symmetry

chain of length $L = 13$ with
N fixed spins up (or down)
$D^{\mathrm{even}} = 651$

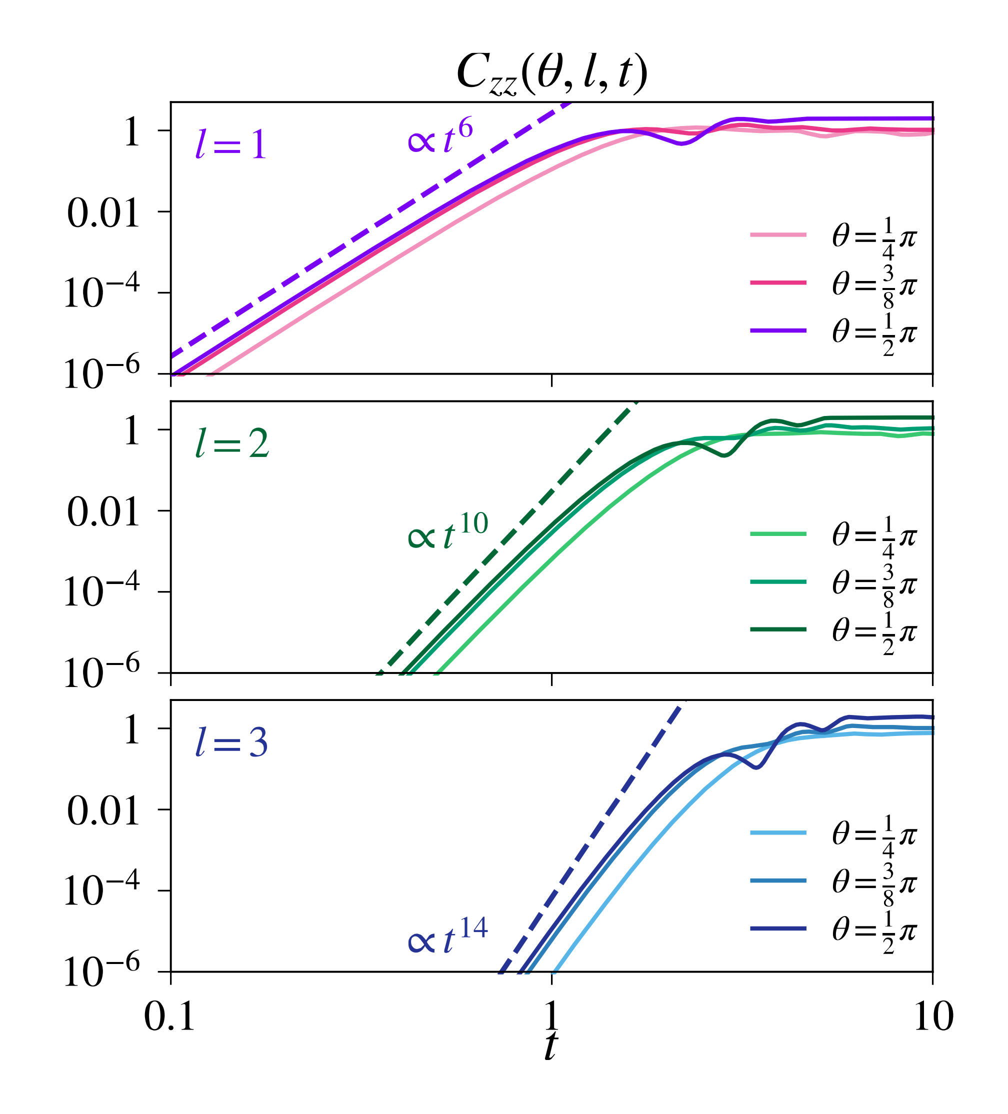


Ising model with tilted magnetic field

$$\hat{H} = J \sum_{i=0}^{L-2} \hat{S}_i^z \hat{S}_{i+1}^z + B \sum_{i=0}^{L-1} \left(sin\left(\theta\right) \hat{S}_i^x + cos\left(\theta\right) \hat{S}_i^z \right)$$

(see e.g. Karthik, Sharma & Lakshminarayan, PRA **75** 022304 (2007))

$$C_{zz}\left(l,t\right) \approx \frac{1}{2} \left(B \sin\left(\theta\right)\right)^{2(2l+1)} \frac{t^{2(2l+1)}}{\left((2l+1)!\right)^2}$$



$\theta = 0, \pi/2$

Integrable, highly degenerate noninteracting fermions

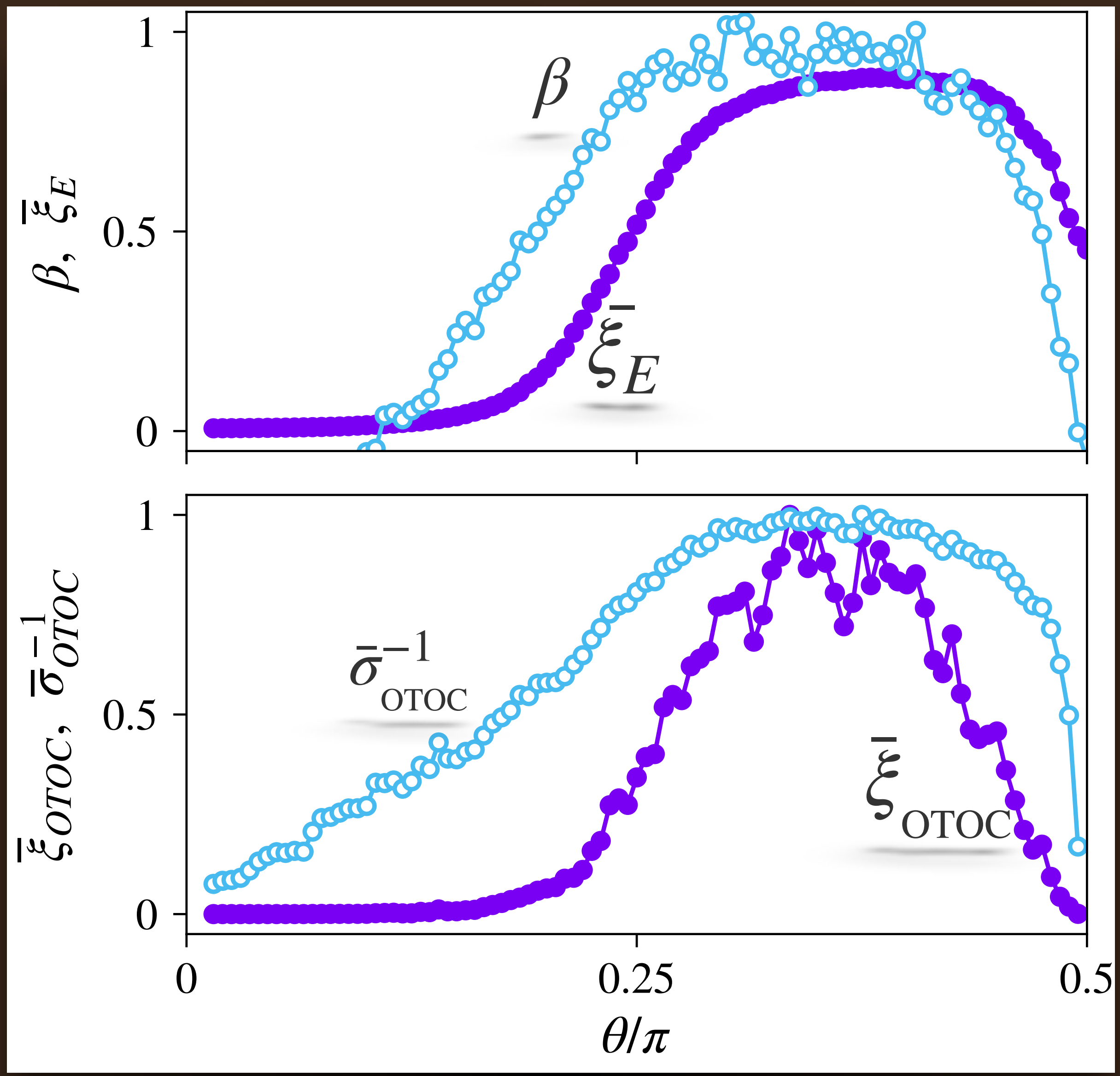
$0 < \theta < \pi/2$

Integrable, highly degenerate interacting fermions (chaos)

see e.g. J. Karthik, A. Sharma, and A. Lakshminarayan, Phys. Rev. A 75, 022304 (2007)

Parity Symmetry

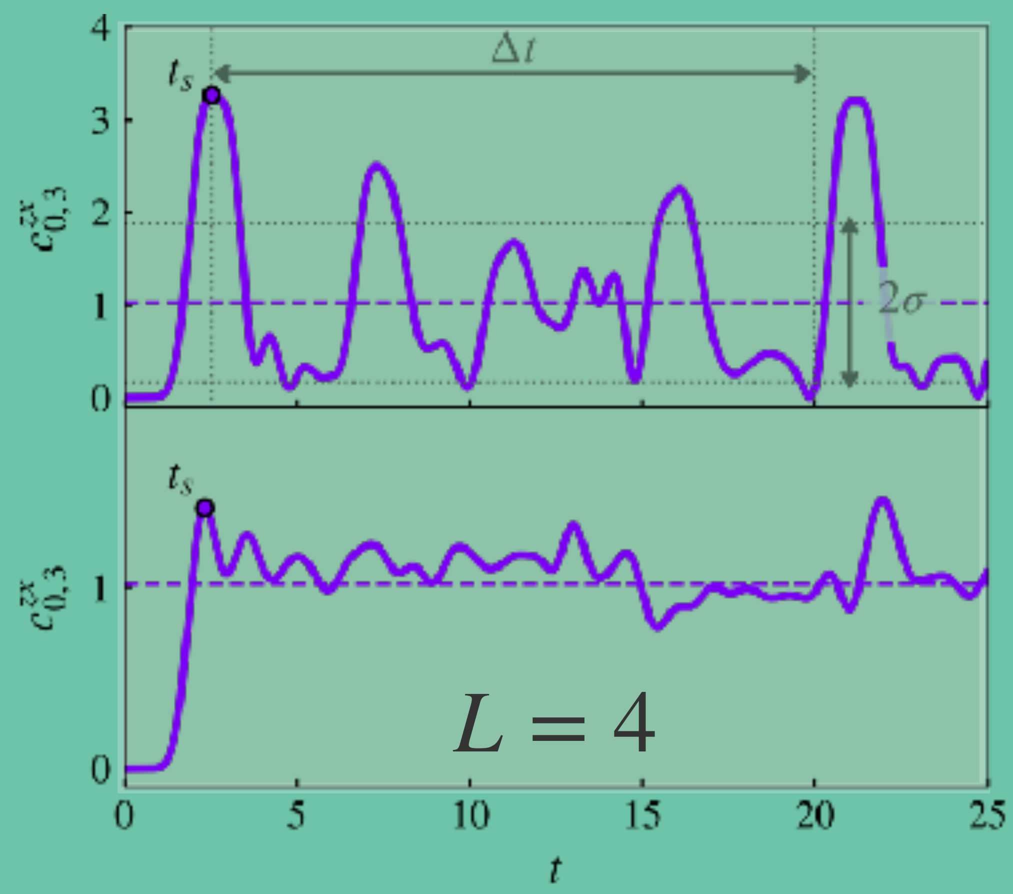
$L = 12$
$D^{\text{even}} = 2080$



Full space $D = 2^L$
$L = 8$

$$\hat{H}\left(J, h_x, h_z\right) = -J \sum_{i=0}^{L-2} \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z + \sum_{i=0}^{L-1} \left(h_x \hat{\sigma}_i^x + h_z \hat{\sigma}_i^z\right)$$

$$c(t) \equiv C(t)/\langle C(t) \rangle$$

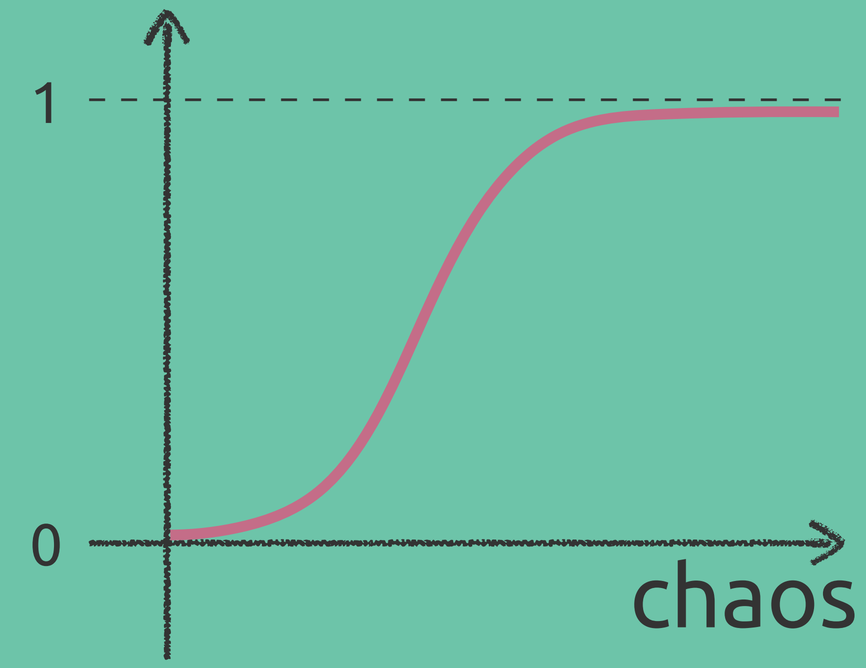


$$C_{ij}^{\mu\nu}(t) = \frac{1}{2} \left\langle \left[\hat{\sigma}_i^\mu(t), \hat{\sigma}_j^\nu \right]^2 \right\rangle$$

$$= 1 - \mathrm{Re}\left\{ \mathrm{Tr}\left[\hat{\sigma}_i^\mu(t) \hat{\sigma}_j^\nu \hat{\sigma}_i^\mu(t) \hat{\sigma}_j^\nu \right] \right\} / D$$

$$\chi = \frac{\sigma^{-1} - \sigma_{\mathrm{min}}^{-1}}{\sigma_{\mathrm{max}}^{-1} - \sigma_{\mathrm{min}}^{-1}}$$

$$\sigma = \sqrt{\langle c(t)^2 \rangle - 1}$$



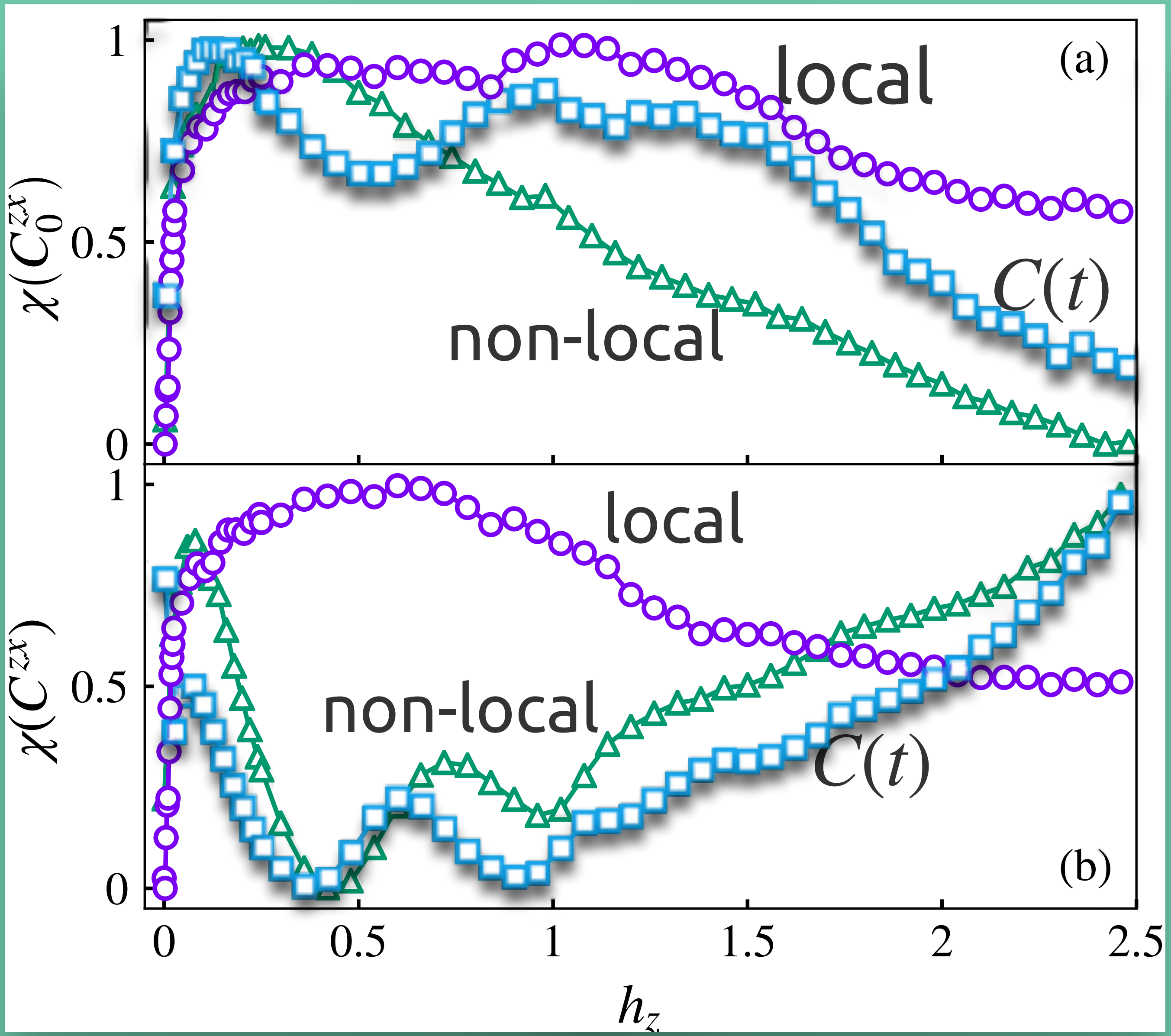
operadores no locales

magnetización total a lo largo de μ $\hat{\sigma}^{\mu} = \sum_{i=0}^{L-1} \hat{\sigma}_i^{\mu}$

$$C(t) = C(t)_{\mathrm{local}} + C(t)_{\mathrm{non-local}}$$

local+global

global+global



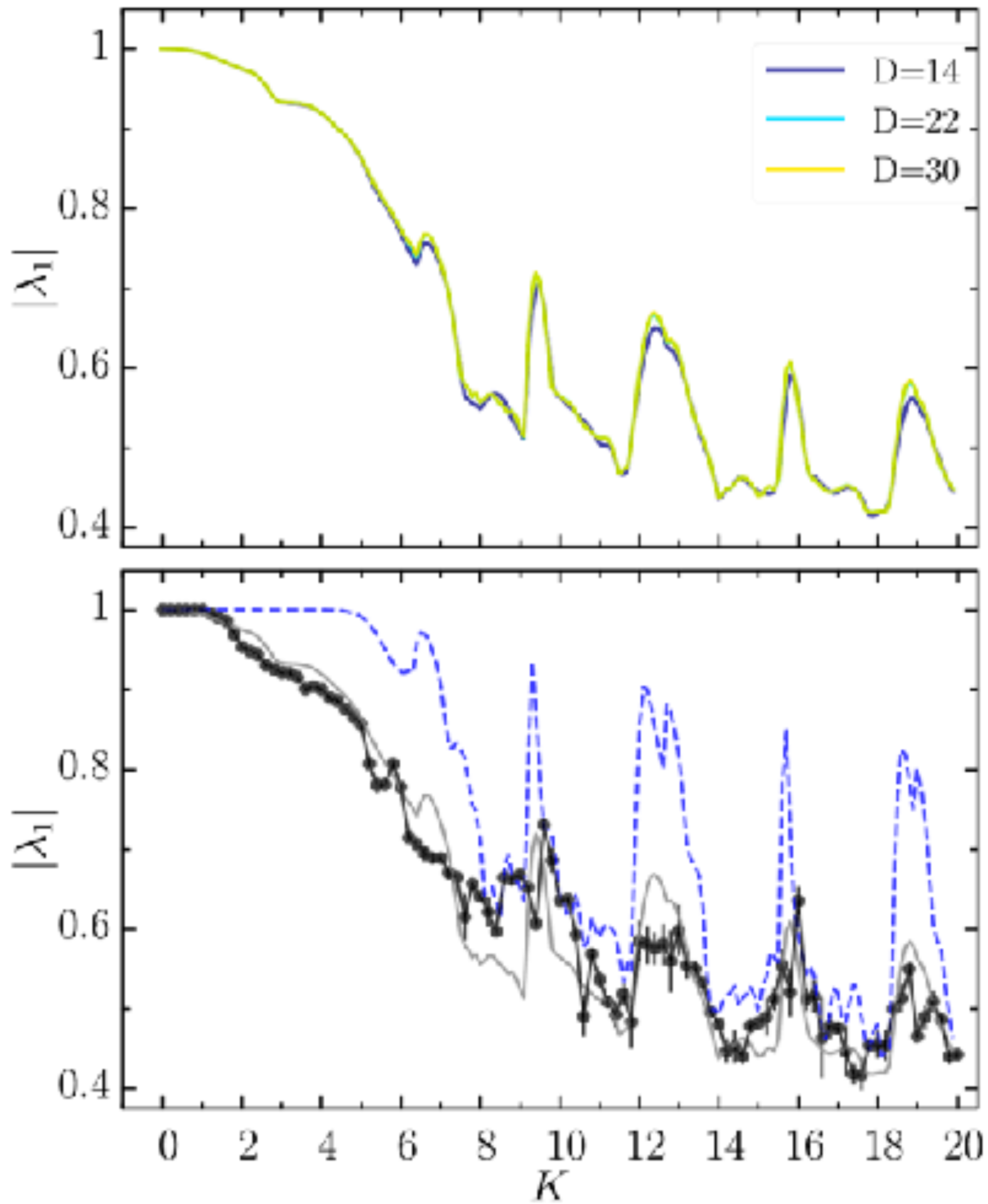


FIG. 8. Top: Eigenvalues $|\lambda_1|$ of the PF operator computed with the Fourier method versus the chaos parameter for increasing values of D and noise $\sigma = 0.2$. Bottom: Eigenvalues $|\lambda_1|$ of the PF operator calculated with the Fourier method without noise (blue dashed lines) and $e^{-\gamma/2}$ obtained from decay rates γ (black dots and solid line) versus the chaos parameter. The eigenvalues calculated with noise $\sigma = 0.2$ are shown by gray lines. The PF discrete matrix has dimension $D = 30$.

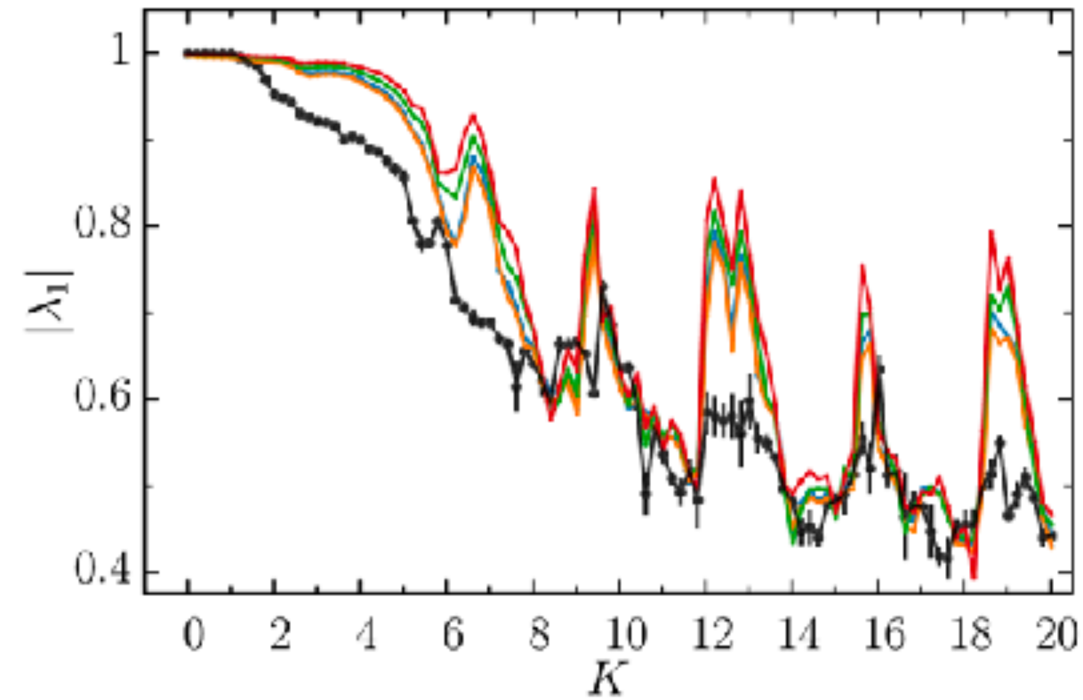


FIG. 9. Ulam eigenvalues versus K for different matrix dimensions D. For these values, $D = 30, 40, 50$ (orange, blue, and green lines, respectively), we show the eigenvalues calculated with the addition of noise with normal distribution $\mathcal{N}(0, \hbar_{\text{eff}})$ with $\hbar_{\text{eff}} = 1/2\pi D$. We also show the eigenvalues for $D = 30$ without noise (red line).