Indistinguishability, individuality and dark energy:

Cosmology, ontology and statistics

Part I: Historical/Philosophical introduction

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Making things clear

- **Identical particles**: have all properties identical (**Philosophy**) – have all **intrinsic** (i.e. non dynamical) properties identical (**Physics**).
- **Indistinguishable particles**: have **all properties** identical (Physics and Philosophy). **Contingent** (state dependent) condition. They possess **cardinality**: the colective state tells the number of particles.
- **Indiscernible particles**: **we cannot tell** the difference (epistemological definition).
- Question to be addressed: ¿what is a (*definite*) **property** in quantum physics?

The impenetrability of classical particles and the Principle of Identity of Indiscernibles

*… it seems to me that God in the Beginning formed Matter in solid, massy, hard, **impenetrable**, moveable Particles….*

I. Newton, *Opticks*, New York: Dover (repr. from 4th ed. (1730)), p. 400.

*Assumption 1. We imagine that two different material points **never occupy the same place at the same time** or come infinitely close together.*

L. Boltzmann, *Principles of Mechanics*, in *Theoretical Physics and Philosophical Problems*, McGuinness, B. (ed.), Dordrecht: Reidel.

If no two particles can occupy the same place, then the PII is **automatically satisfied**.

Individuality, identity and countability

an object that is differentiated from others of its kind in such a fashion that it and they are ***apt to constitute a countable plurality****, with each member of such a plurality counting for* ***just one****, a unit of its kind…*

… the items to be counted should possess ***determinate identity conditions****, since each should be counted just once and this presupposes that it be* ***determinately distinct*** *from any other item that is to be included in the count.*

Lowe, E. J., *Primitive Substances*, *Philosophy and Phenomenological Research* **54**, pp. 531–52 (1994).

Edward Jonathan Lowe

(Dover, England, 1950 – 2014)

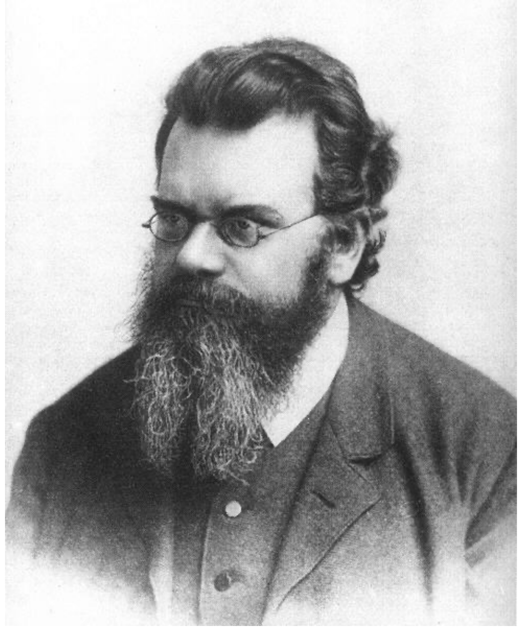
¿Anticipating the energy-matter (statistical) equivalence?

*I see no reason why energy shouldn't also be regarded as **divided atomically**.*

Boltzmann, *Halle Conference*, 1891.

*One could even calculate the possibilities of the various states from the ratios of the **number of ways** in which their distributions could be achieved, which could perhaps lead to an **interesting method** of calculating thermal equilibrium.*

Boltzmann, L. (1868), *Studien über das Gleichgewicht der lebendingen Kraft zwischen bewegten materiellen Punkten**, *Wiener Berichte* **58**, pp. 517–60.



Ludwig Eduard Boltzmann

(Viena, 1844 – Duino, Italia, 1906)

Measuring the permutability

… the total ***permutability measure*** *of all the bodies will increase continuously during the* ***change of state*** *and it can at most remain constant if all the bodies throughout the transformation approximate* ***thermal equilibrium infinitely closely****…*

(1905), *Populäre Schriften*, McGuinness (ed.), Dordrecht: Reidel.

* *Estudios sobre el equilibrio de la fuerza viva entre puntos materiales en movimiento*.

Space-Time Individuality

*… our procedure in regarding the interchange of two similar molecules as corresponding to a **significant change in the mechanical state** of a system, even though not in its condition, evidently implies the possibility of **keeping a continuous observation on the system** which would let us know whether two similar molecules change roles or not. This, however, is in entire agreement with the point of view of the classical mechanics, which would permit such a continuous observation, at least in principle…*

Tolman, R. (1962), *Principles of Statistical Mechanics*, Oxford: Oxford University Press (1938), p.77.

Richard Chace Tolman, West Newton, USA, 1881 - 1948

Trans-temporal identity and material genidentity in classical physics

… we find that whenever two material objects exchange their spatial positions this fact is noticeable. We usually recognise this change of position by the use of ***specific marks on the objects****…These marks remain on the object in accordance with the* ***continuity criterion*** *and permit an identification of the objects even when no observation during the change of spatial positions was made and the continuity criterion cannot be applied. In other words,* ***an interchange of spatial positions is a verifiable change*** *even though no records of the act of interchanging are available.*

Reichenbach, H. (1956), *The Direction of Time*, Berkeley: University of California Press.

Hans Friedrich Herbert Günther Reichenbach, Hamburgo (Imperio alemán), 1891 - Los Angeles, USA, 1953

¿Are all the points of space-time the same point or, is the PII aplicable to space-time points?

The distinguishing feature of a particular point of… space-time is that it has no distinguishing features; ***all points of space-time are assumed to be equivalent****.*

J. L. Anderson, *Principles of Relativity Physics*, Academic Press (1967).

James Leroy Anderson
Chicago, 1926 - Falmouth, Massachusetts, 2021

¿What is the space-time manifold?

*… if we ask (assuming Newtonian physics) whether 'equality of time-intervals' is a relation intrinsic to the space-time manifold, and if this is construed (roughly) to mean 'whether that relation is involved in the structure of the spacetime manifold itself, **considered apart from all other entities**', the question at once arises of how to explicate the notion of '**the space-time manifold itself** ', and of the conceptual line between it and 'all other entities'. I see no way to confront the former question independently of the latter; and yet the converse may also seem to hold: that we cannot give a conceptual explication of 'the space-time manifold' **without begging the question** of its intrinsic properties.*

H. Stein, cited by **Adolf Grünbaum** in his article *Absolute and Relational Theories of Space and Space-Time*, in *Foundations of Space-Time Theories*, J. Earman, C. Glymour and J. Stachel (eds.), Minneapolis, MN: University of Minnesota Press, p. 329, (1977).

Howard Stein

New York City, 1929 – Chicago, 2024

The space-time points: *¿are they or not?*

This prompts my question about ***individuation of world-points****, which is addressed to both the all-out space-time absolutist and to his relationalist critics of one stripe or another. The question is: what criteria of identity or distinctness for world-points and/or punctal events, if any, can give physical meaning to the required* ***formal disidentifications at the ontological level*** *of postulated space-time theory? This question is* ***logically prior*** *to the further hard question of whether such disidentifications can also be legitimated empirically at the epistemological level, rather than merely at the level of postulational meaningfulness.*

Adolf Grünbaum, *Absolute and Relational Theories of Space and Space-Time*, in *Foundations of Space-Time Theories*, J. Earman, C. Glymour and J. Stachel (eds.), Minneapolis, MN: University of Minnesota Press, p. 366, (1977).

A. Grünbaum: Cologne (Weimar Republic), 1923 - Allegheny County (Pensilvania, USA), 2018

¿Substance without identity ?

Space-time points have no duration, and hence no trajectories over time. They also ***do not interact in any way with each other****, or with physical objects or fields that occupy them. If they did have duration, trajectories, and nontrivial interactions with other points or objects, then they might be capable of the kind of statistical behavior seen in the dice: so-called Maxwell-Boltzmann statistics. This would not mean that space-time points have primitive identity, however: we would not have to grant that it makes sense to suppose that point A had the trajectory and qualitative history that B actually has, and vice versa.* ***What goes for points, goes for particles****. Even if atoms had distinct and continuous trajectories,* ***we would not have to ascribe primitive identity to them in order to think of them as real substances****. The ascription of primitive identity allows us to pose certain strange philosophical questions - but not to do any more productive work.*

Carl Hoefer, (1996), *The Metaphysics of Spacetime Substantivalism*, Journal of Philosophy **93**, pp. 5–27.

Two ***different*** postulates

Indistinguishable Postulate: there is no **measurement** that can detect the difference between permuted (final) and unpermuted (initial) states→ **restriction on operators** describing observables:

$$< \psi|\hat{O}|\psi > = < \hat{P}\psi|\hat{O}|\hat{P}\psi > = < \psi|P^{-1}\hat{O}\hat{P}|\psi >$$

$$\rightarrow \ [\hat{O}, \hat{P}] = 0.$$

Symmetrization Postulate: only completely symmetrized or completely anti symmetrized states are allowed. **Restriction** on the (accesible) **Hilbert space of states**.

Relational definitions

Bosons: particles whose wave function is **completely symmetric**.

Fermions: particles whose wave function is **completely antisymmetric**.

These definitions do not include, *a priori*, any mention to **intrinsic properties such as *spin***.

The spin-statistics connection does not preclude intermediate statistics

Quantum field theories of spin ½ particles cannot lead to **symmetric** wave functions, as it obtains **negative probabilities**. That **does not** mean that spin ½ particles **must** be fermions, but only that they **cannot** be bosons.

In a quantum theory of particles with integer spin, any **antisymmetric** wave function (*i.e.* fermions) has **fields that are always zero**. That **does not** mean that integer spin particles **must** be bosons, but only that they **cannot** be fermions.

The birth of wave-particle duality

Louis-Victor de Broglie (Dieppe, Francia, 1892 - Louveciennes, Francia, 1987): first physicist to propose that **material particles** can be described as **waves**.

Compres Rendus 177, 507, 548, 630 (1923); *Nature* 112, 540 (1923); *Thèse de doctorat* (Masson et Cie, Paris, 1924); *Annales be Physique* 3, 22 (1925) (reimpressed in english in *Wave Mechanics*, ed. by G. Ludwig, (Pergamon Press, New York, 1968); *Phil. Mag.* 47, 446 (1924).

Werner Heisenberg: Wurzburgo, Alemania, 1901 - Múnich, 1976

Positivism and matrix mechanics

*Many of the **abstractions** that are characteristic of modern theoretical physics are to be found discussed in the philosophy of the past centuries. At that time, these abstractions could be disregarded as mere **mental exercises** by those scientists whose only concern was with reality, but today we are compelled by the refinements of experimental art **to consider them seriously**.*

W. Heisenberg, *The Physical Principles of the Quantum Theory,* Chicago, IL: Chicago University Press (1930).

Discontinuity and individuality

*Notwithstanding the difficulties which hence are involved in the formulation of the quantum theory, it seems, as we shall see, that its essence may be expressed in the so-called **quantum postulate**, which to any atomic process attributes an essential **discontinuity** or rather **individuality**, completely foreign to the classical theories and symbolised by **Planck's quantum of action**…*

Niels Henrik David Bohr, Copenhaguen, 1885 - 1962

Isolation as indisponsable abstraction

*… it must be kept in mind, that according to the view taken above, radiation in free space as well as isolated material particles **are abstractions**, their properties on the quantum theory being observable and definable **only through their interactions** with other systems. Nevertheless these abstractions are… **indispensable** for a description of experience in connection with our ordinary space-time view.*

Niels Bohr's Collected Works, edited by J. Kalckar, Amsterdam: North-Holland (1985, paper published in *Nature*, 1928).

Individuality, complementarity, discontinuity and unity

… the apparent contrast between the ***continual onward flow*** *of associative thinking and the preservation of the* ***unity of the personality*** *exhibits a suggestive analogy with the relation between the* ***wave description*** *of the motions of material particles, governed by the superposition principle, and their* ***indestructible individuality****.*

The Quantum of Action and the Description of Nature, in **N. Bohr**, *Atomic Theory and the Description of Nature*, Cambridge: Cambridge University Press, pp. 92–101 (1929).

The Superposition Principle as reciprocal to individuality

*Thus one can pursue the reciprocity of the concept of individuality and the superposition principle to **far-reaching consequences**. One can show in general that any use of the former concept limits the application of the latter principle as an immediate consequence of the loss of phase resulting from every observation. Conversely, any consistent application of the superposition principle **limits the possibility of a visualizable interpretation based on the principle of individuality**, as it finds expression above all in the quantum theoretical treatment of **systems with several identical particles**. All this contains of course nothing really new.*

Bohr in a letter to **Pauli** (1929), in N. Bohr, *Atomic Theory and the Description of Nature* (Cambridge University Press, Cambridge, 1934).

Julius Robert Oppenheimer: New York City, 1904 - Princeton, New Jersey, 1967

A farewell to permanence and identity

We are continuing the attempt to discover, to identify and characterize, and surely ultimately to order, our knowledge of ***what the elementary particles of physics really are****. I need hardly say that in the course of this we are learning again* ***how far*** *our notion of elementarity, of what makes a particle elementary, is from the* ***early atomic ideas of the Hindu and Greek atomists****, or even from the* ***chemical atomists*** *of a century ago. We are finding out that what we are forced to call elementary particles retain* ***neither permanence nor identity****, and they are elementary only in the sense that their properties cannot be understood by breaking them down into subcomponents.*

R. Oppenheimer (1947), *Physics in the Contemporary World*. Lecture given at MIT, reproduced in M. Gardner (ed.), *The Sacred Beetle and Other Great Essays in Science*, New York: New American Library, 1984, pp. 198–214.

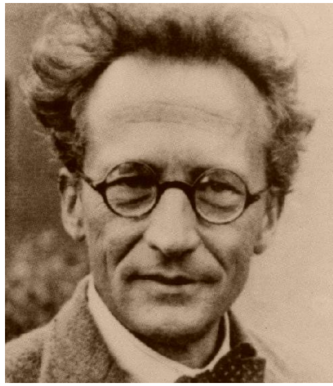
Erwin Schrödinger: Erdberg, Viena, 1887 – Viena, 1961

The meaning of sameness

[t]he circumstances may be such that they render it highly convenient and desirable to express oneself so, but it is only an abbreviation of speech; for there are other cases where the 'sameness' becomes entirely meaningless; and ***there is no sharp boundary****, no clear-cut distinction between them, there is a gradual transition over intermediate cases. And I beg to emphasize this and I beg you to believe it:* ***It is not a question of our being able to ascertain the identity*** *in some instances and not being able to do so in others. It is beyond doubt that the question of 'sameness', of identity,* ***really and truly has no meaning***.

Science and Humanism, Cambridge: Cambridge University Press (1952).

Quantum statistics to particles, Boltzmann statistics to waves… ¿the true individuals?



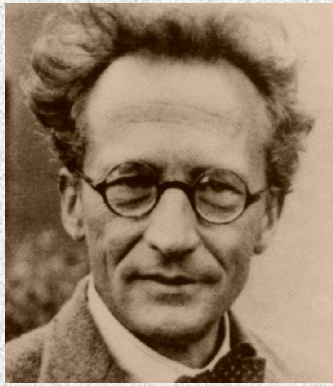
*This means **much more** than that the particles or corpuscles are all alike. It means that you must **not even imagine** any one of them to be marked 'by a red spot' so that you could recognize it later as the same…*

*waves can easily be marked, by their **shape or modulation**…*

*You know that if you deal with such many-body problems as the He-molecule or a gas of 10^{20} molecules, by the method of wave-mechanics, the **proper modes have to be regarded as distinguished from one another**, they have to be treated as **true individuals**. You must not apply Fermi-Dirac statistics or Bose-Einstein statistics to them, but ordinary Boltzmann statistics: then you obtain the correct results, the same as you get by applying the newfangled statistics to the non-individual corpuscles.*

The Interpretation of Quantum Mechanics: Dublin Seminars (1949–1955) and other unpublished essays, edited with introduction by Michel Bitbol, Woodbridge, Conn.: OxBow Press (1995).

Observables constitute individuality



The way in which particles constitute matter is thus a very strange and novel one. We must investigate it more closely. ***Particles, having no individuality, constitute pieces of matter that have*** (individuality). *They do it by* ***giving rise to observables****. What we usually call the building material is of a* ***fundamentally different nature*** *from what is built up of it. In current quantum mechanics, this different nature is expressed by the twofold set of mathematical entities we use: vectors and tensors, or wave functions and operators…*

(individual particles) ***coalesce*** *to form a more extended gathering of a build or constitution* ***not copied in the immediate neighbourhood*** *by a number of similar gatherings of the same build.*

Íbid.

The electron is guilty, no matter who (it) is

… the possibility that one of the identical twins Mike and Ike is in the quantum state E1 and the other in the quantum state E2 does not include two differentiable cases which are permuted on permuting Mike and Ike; ***it is impossible for either of these individuals to retain his identity*** *so that one of them will always be able to say 'I'm Mike' and the other 'I'm Ike.' Even in principle one cannot demand an alibi of an electron!*

The Theory of Groups and Quantum Mechanics, London: Methuen and Co.; English trans. 2nd ed. (1931)

Hermann Weyl: Elmshorn, Germany, 1885 - Zúrich, Switzerland, 1955

Parastatistics: the colective-individual (dis) connection

- States that are **neither symmetric nor antisymmetric** under permutation of identical particles.
- Associated with **higher dimensional representations** of the permutation group.
- Discussed by **Dirac** and **Pauli**, among others.
- **No experimental evidence** yet.
- If the Hamiltonian is **permutation symmetric**, once a state has a given permutation symmetry, it keeps having it (remains in **the same subspace**).
- No reason (yet) to associate any **collective** state symmetry property with **individual** properties→***holistic approach*** (Hilborn & Yuca, 2002).

Entanglement, exclusion and identity

$$\psi(1,2) = \frac{1}{\sqrt{2}}[\phi_a(1)\phi_b(2)\text{-}\phi_b(1)\,\phi_a(2)]$$

Entanglement: **sum** of two terms.

Exclusion: **minus** sign; if a=b the total wave vanishes.

Identity: **same** intrinsic properties.

(Aparent?) Paradox: the **density matrices** of both particles are the same

$$\rho(1) = \rho(2) = \frac{1}{2}[\phi_a\rangle\langle\,\phi_a + \phi_b\rangle\langle\,\phi_b]$$

¿PII violated?
Indiscernible quantum particles are countable: there can be more tan one

- PII holds, but identical particles are distinguished by some **unknown and quantum irrelevant** property.
- Quantum particles are individuals (**Trascendental Individuality***?) but PII is violated.
- Quantum particles are **not** individuals and PII is **not** violated.

*unobservable "thing" that confers the particle identity and **capacity to bear labels**.

Particle ontology in Quantum Field Theory

- Particles are not individuals, but **excitations** of the quantum field→ **Fock space**.
- **Wave-particle** duality ← → **non-individual-individual** duality.
- ¿**What** do particle creator operators **create**?
- Does the **order of creation** label the particles?
- E.g. fermions: $a_b a_c + a_c a_b = 0 = a^\dagger_b a^\dagger_c + a^\dagger_c a^\dagger_b \Longrightarrow$ if $b \neq c$ then

$$a^\dagger_b a^\dagger_c |0\rangle = |1_c 2_b\rangle = -|1_b 2_c\rangle = a^\dagger_c a^\dagger_b |0\rangle$$

→ antisymmetric state.

A conservation law for statistics

If the particles are indistinguishable, in particular the Hamiltonian **does not** distinguish them, i.e.,

$$[\hat{\mathcal{H}}, \hat{P}] = 0$$

and the same goes for the evolution operator U=exp(-i$\mathcal{H}$t)= $\hat{P}^{\dagger} U \hat{P}$.

Consequently, a state with a given symmetry **cannot evolve into a state of a different symmetry**.

For instance, if $|\psi_s>$ is a symmetric state ($\hat{P}\,|\psi_s>=|\psi_s>$) and $|\psi_a>$ an antisymmetric state ($\hat{P}\,|\psi_a>= -|\psi_a>$), then:

$$< \psi_s |U|\psi_a > = < \psi_s |U|\,\hat{P}\,\psi_a > = - < \hat{P}\psi_s |U|\,\hat{P}\,\psi_a >$$
$$= - < \psi_s |\hat{P}^{\dagger}\,U\hat{P}|\,\psi_a > = - < \psi_s |U|\,\psi_a >$$
$$\Longrightarrow < \psi_s |U|\psi_a > = 0,$$

i.e. **the transition probability is zero**.

¿Are non standard statistics out there?

There are alternative relations between creation and anihilation operators, such as **trilinear** (**Green**, 1952), **Q-mutators** (**Greenberg**, 1990), etc.

Possible tests (Greenberg, 1991):

- **Transitions** between forbidden states (e.g. non antisymmetric states for electrons; symmetric states for electrons if spin-statistics theorem is violated).
- **Accumulation** of particles in forbidden states.
- **Thermodynamic deviation** of (macroscopic) states from Fermi-Dirac or from Bose-Einstein statistics.

No observational evidence yet… ¿yet?

¿Could dark energy be such an evidence?

To be continued...