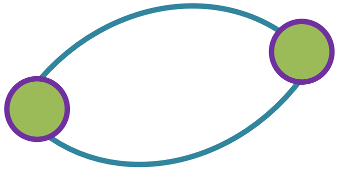
Cascada de fallas en redes complejas

Grupo Sistemas Complejos

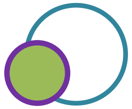
Definición de Red Compleja

Nodos: personas, computadoras, páginas webs, aeropuertos, animales.

Enlaces: representan algún tipo de interacción entre los nodos: amistad, conexiones físicas, vuelos.

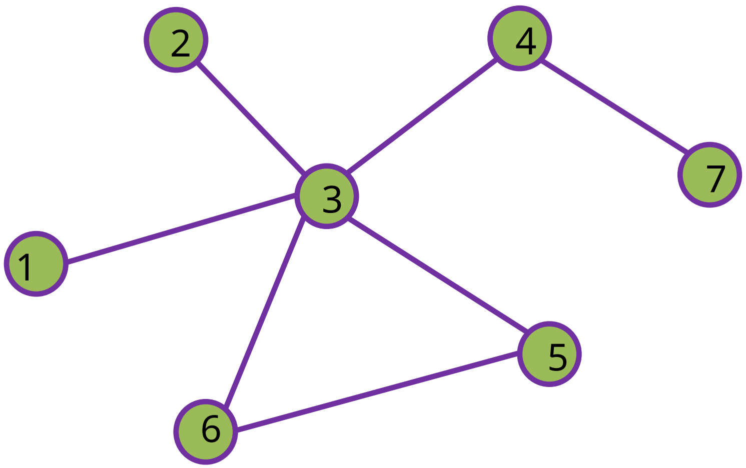


Múltiple conexión

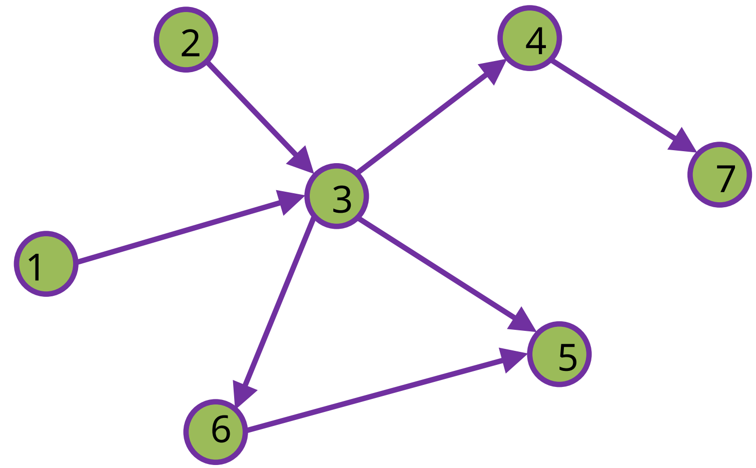


Auto conexión

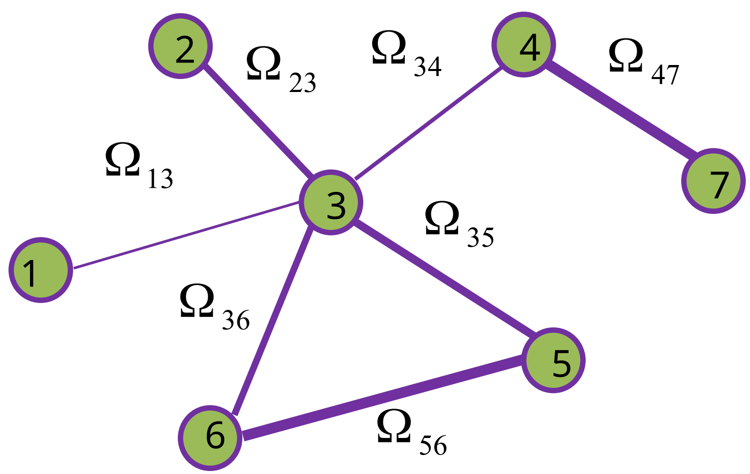
No dirigida

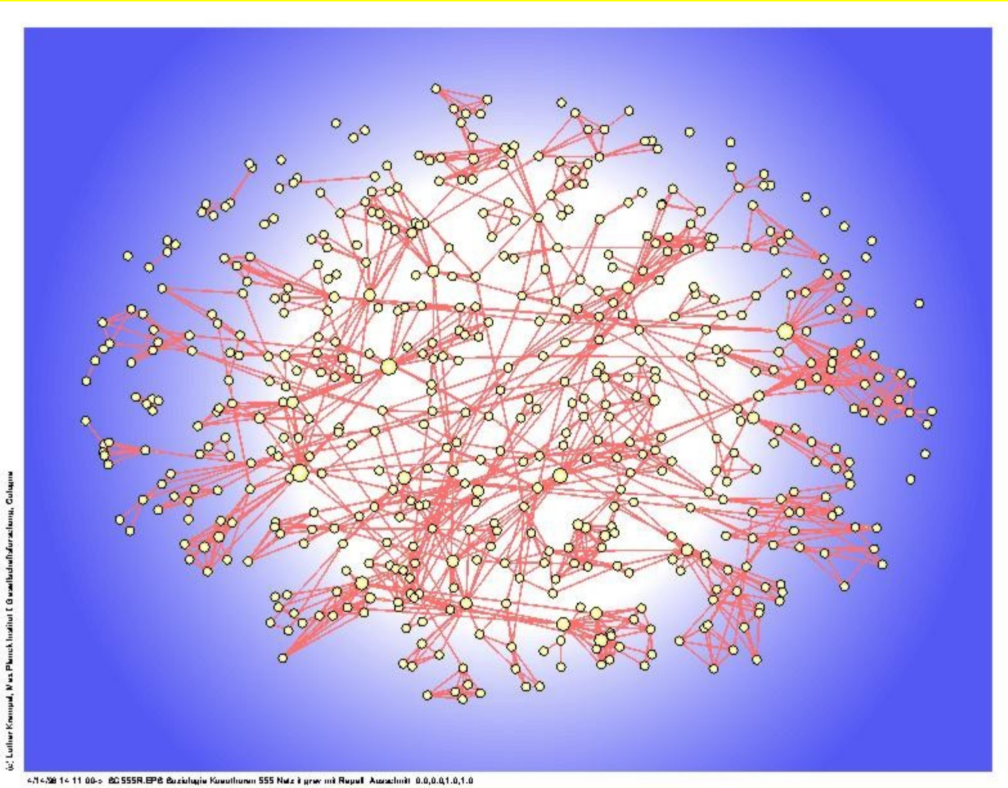


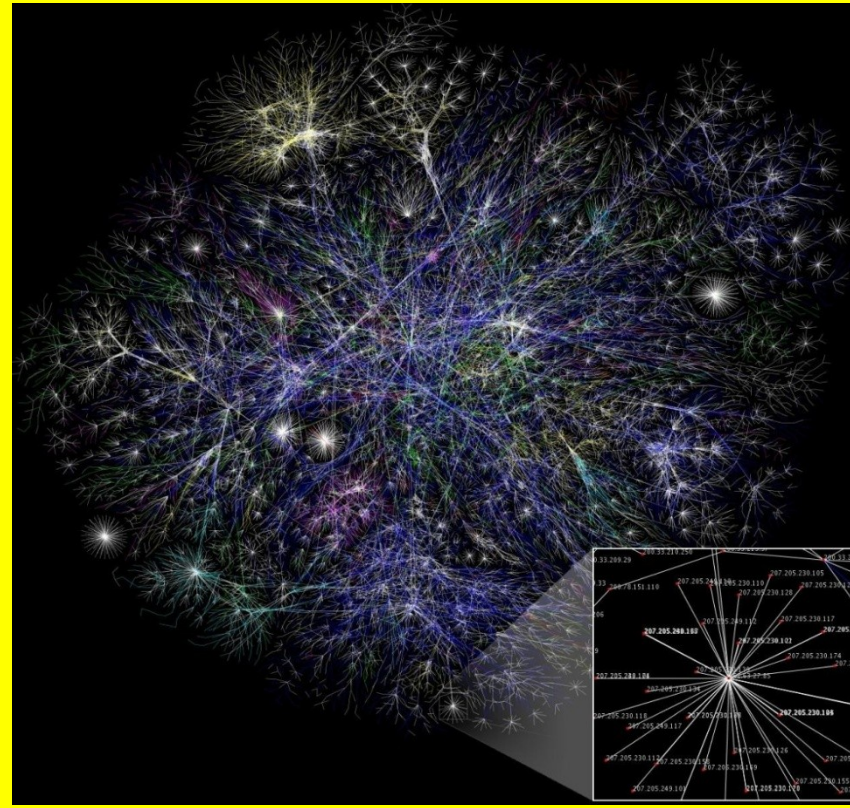
Dirigida



Pesada

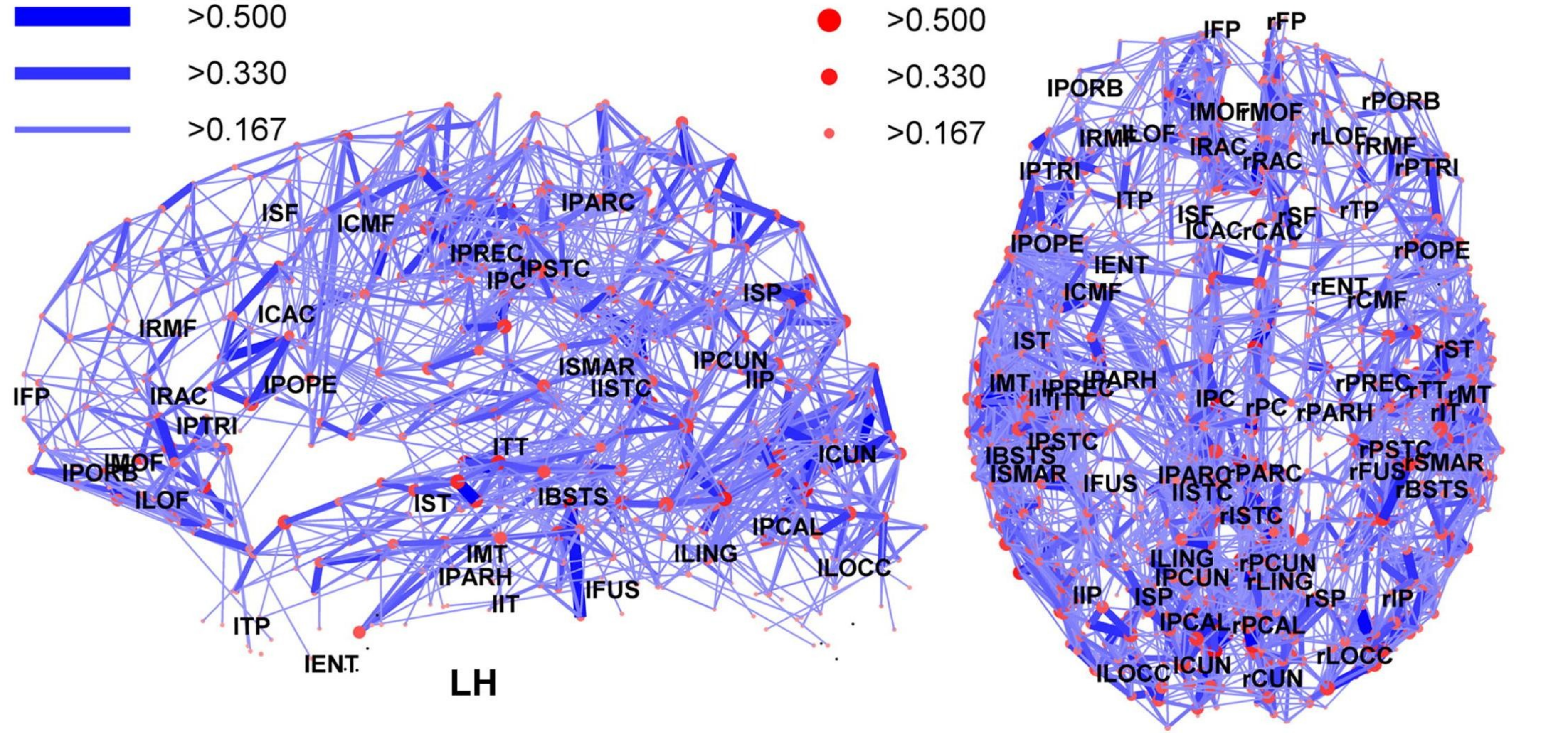






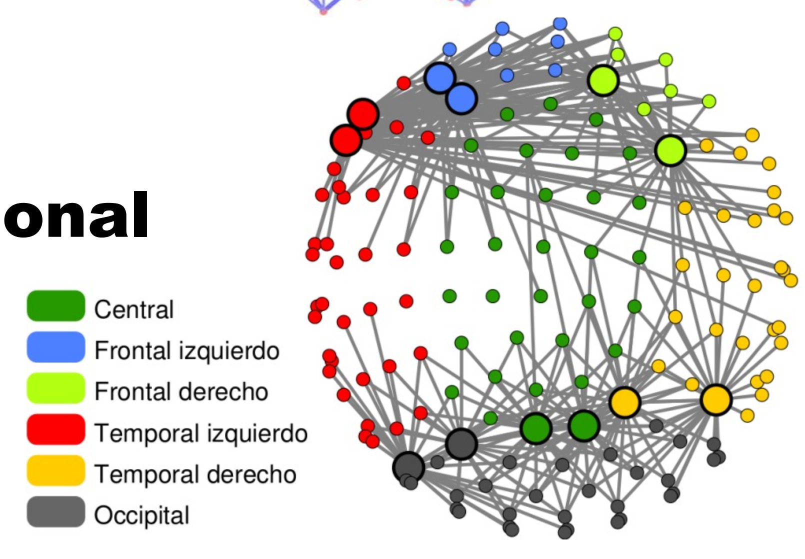
Red de Científicos

Internet



Red cerebral

Red Funcional con DCL



J.M. Buldú, R. Bajo, F. Maestú, N. Castellanos, I. Leyva, P. Gil, I. Sendiña-Nadal, J. A. Almendral, A. Nevado, F. del Pozo and S. Boccaletti, PLoS ONE 6(5)

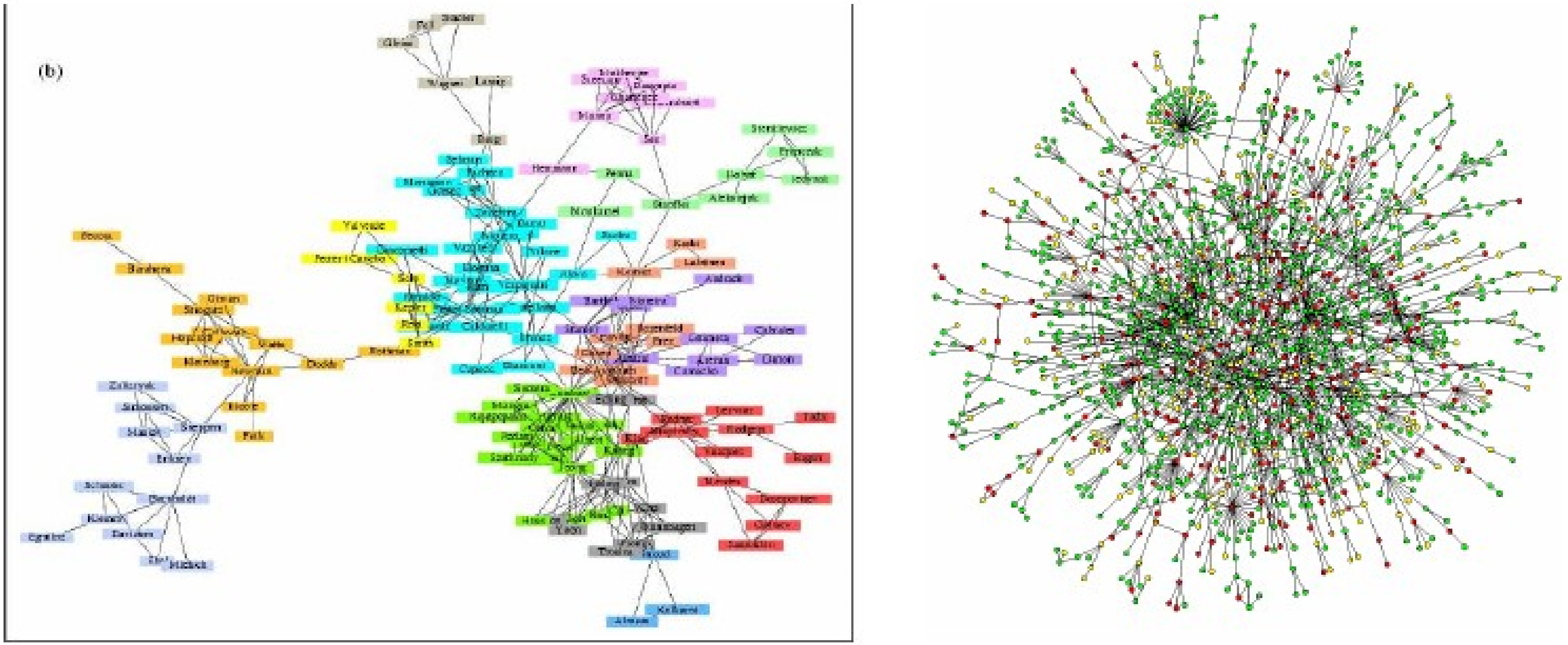


Figura 1.2: **Izquierda:** Redes de Colaboraciones de los grupos que trabajan en redes complejas. (M. E. J. Newman and M. Girvan, *Finding and evaluating community structure in networks*, Physical Review E **69**, 026113 (2004)). **Derecha:** Mapa de las interacciones proteína-proteína. Los colores de cada nodo representan el efecto fenotípico de remover la proteína correspondiente (rojo, letal; verde, no letal; naranja, crecimiento lento; amarillo, desconocido). Por Hawoong Jeong, Notre Dame University

Propiedades generales

Matriz de Adyacencia

$$A_{ij} = \begin{cases} 1 & \text{si } ij \text{ estan conectados} \\ 0 & \text{si } ij \text{ no estan conectados} \end{cases}$$

$$A_{ij} = A_{ji}$$

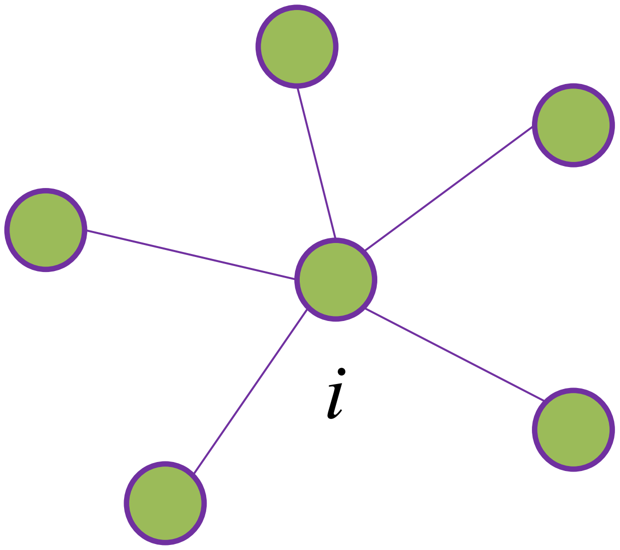
No dirigida

$$A_{ij} \neq A_{ji}$$

Dirigida

Distribución de grado

Grado o conectividad



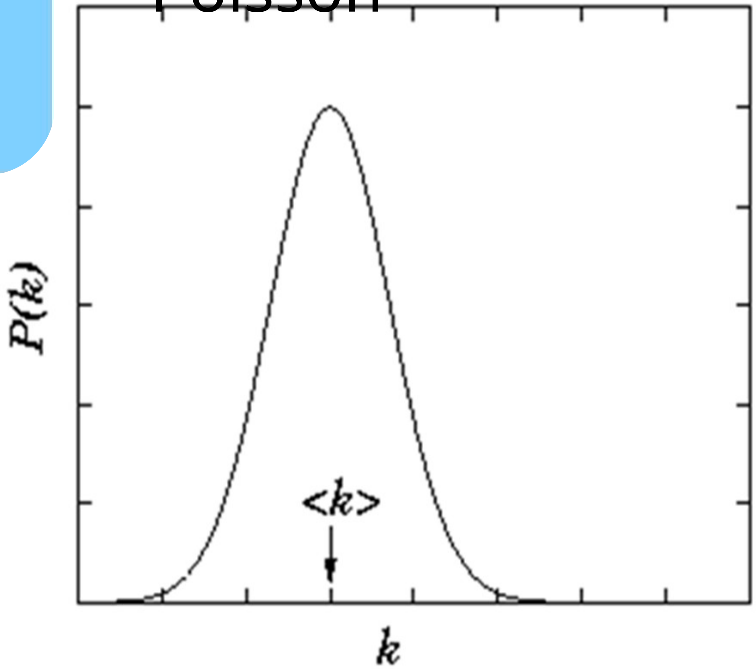
$$k_i = \sum_{j=1}^{N} A_{ij}$$

$$k_i = 5$$

$$P(k) \longrightarrow \langle k \rangle = \sum_{i=1}^{N} k_i P(k)$$

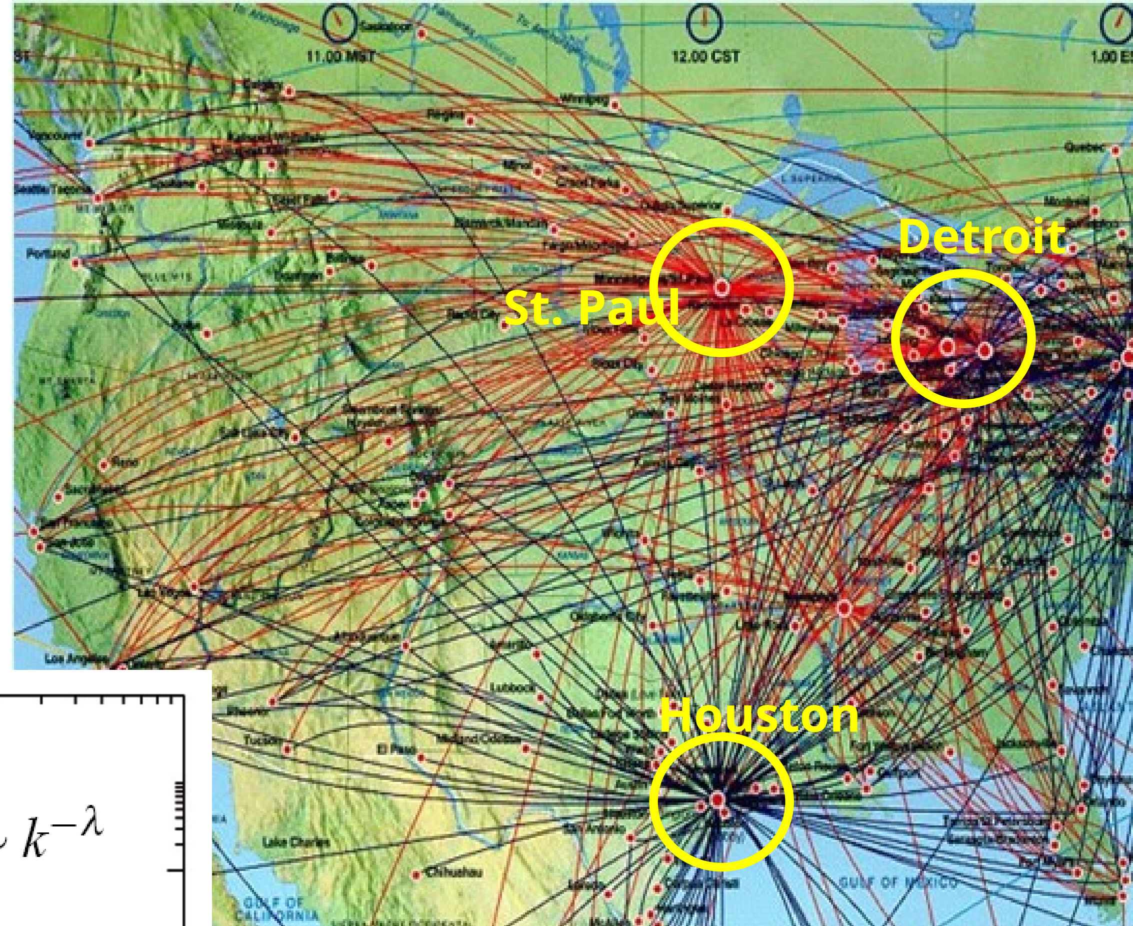
Carreteras de EE.UU.

Distribución de Poisson

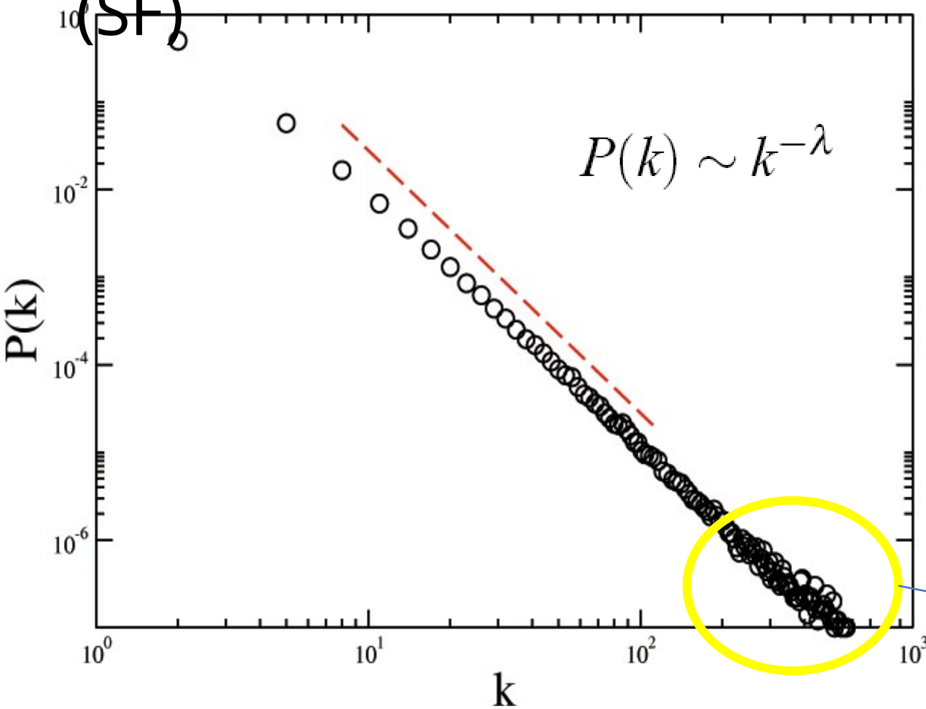


Red Homogénea

Rutas aéreas de EE.UU.

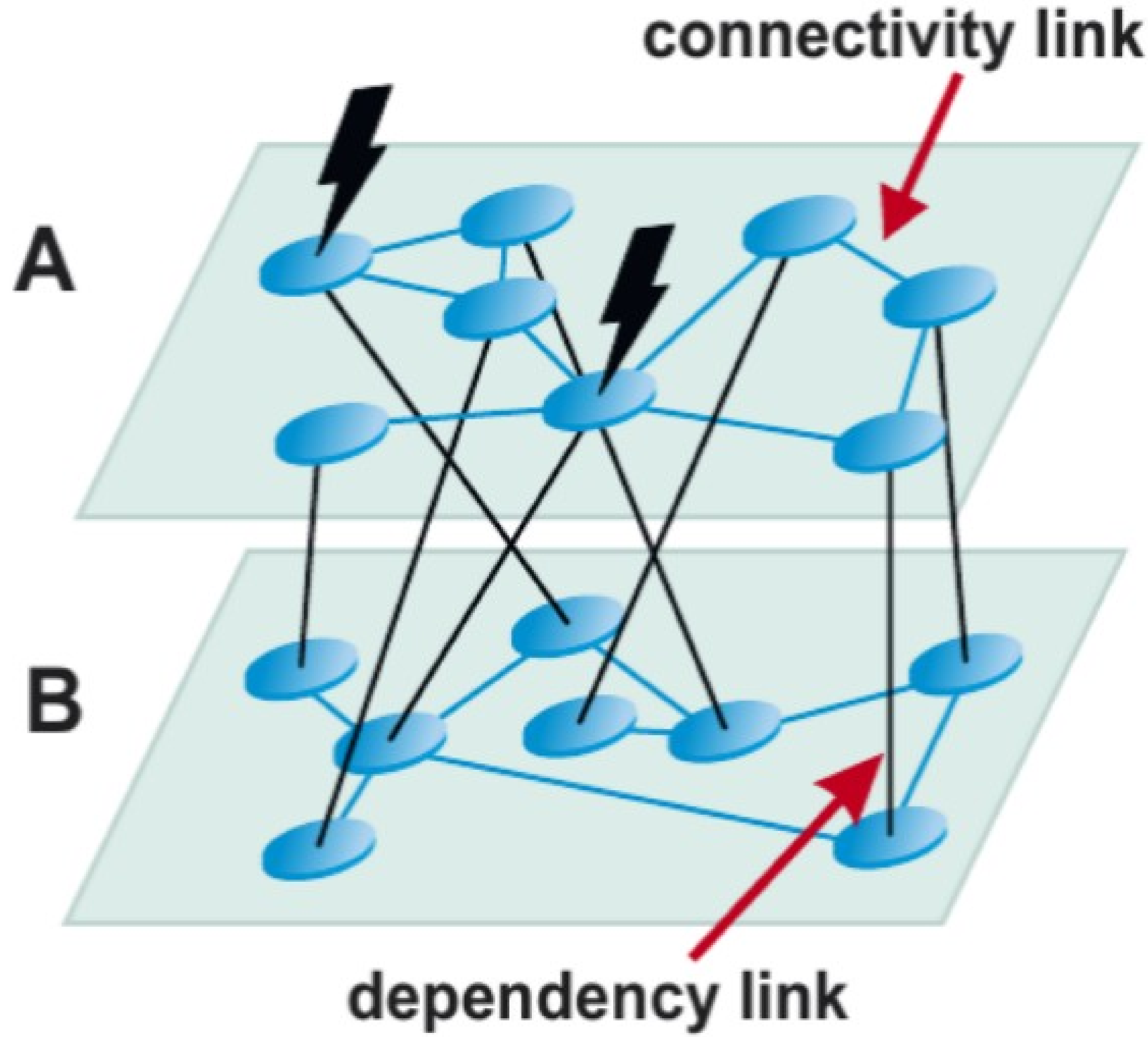


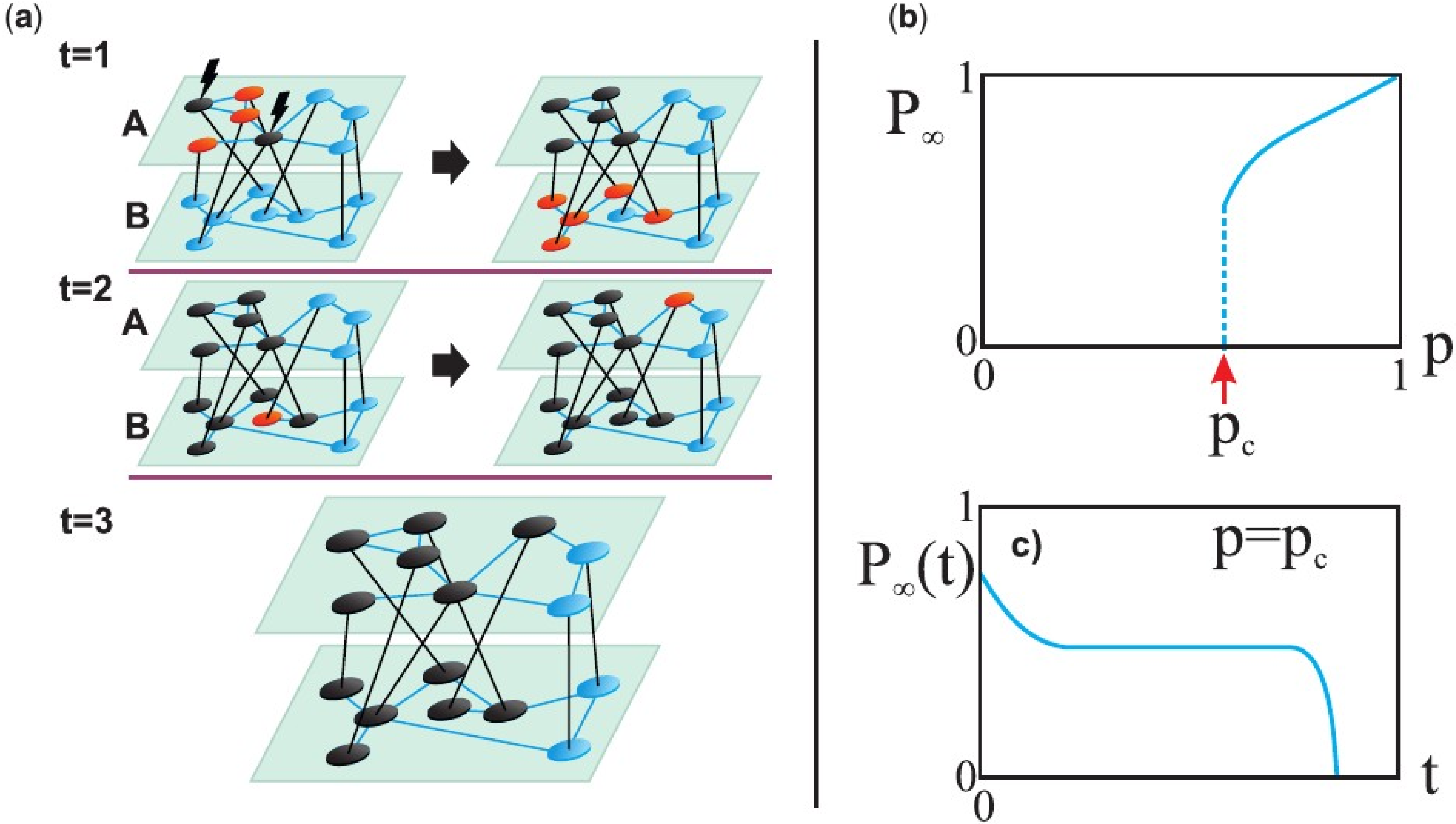
Red Libre de Escala (SF)



Red Heterogénea

hub


connectivity link
A
B
dependency link



(a)
t=1
A
B
t=2
A
B
t=3
(b)
P_∞
1
0
0
1
p
p_c
1
$P_\infty(t)$
c)
$p=p_c$
0
0
t

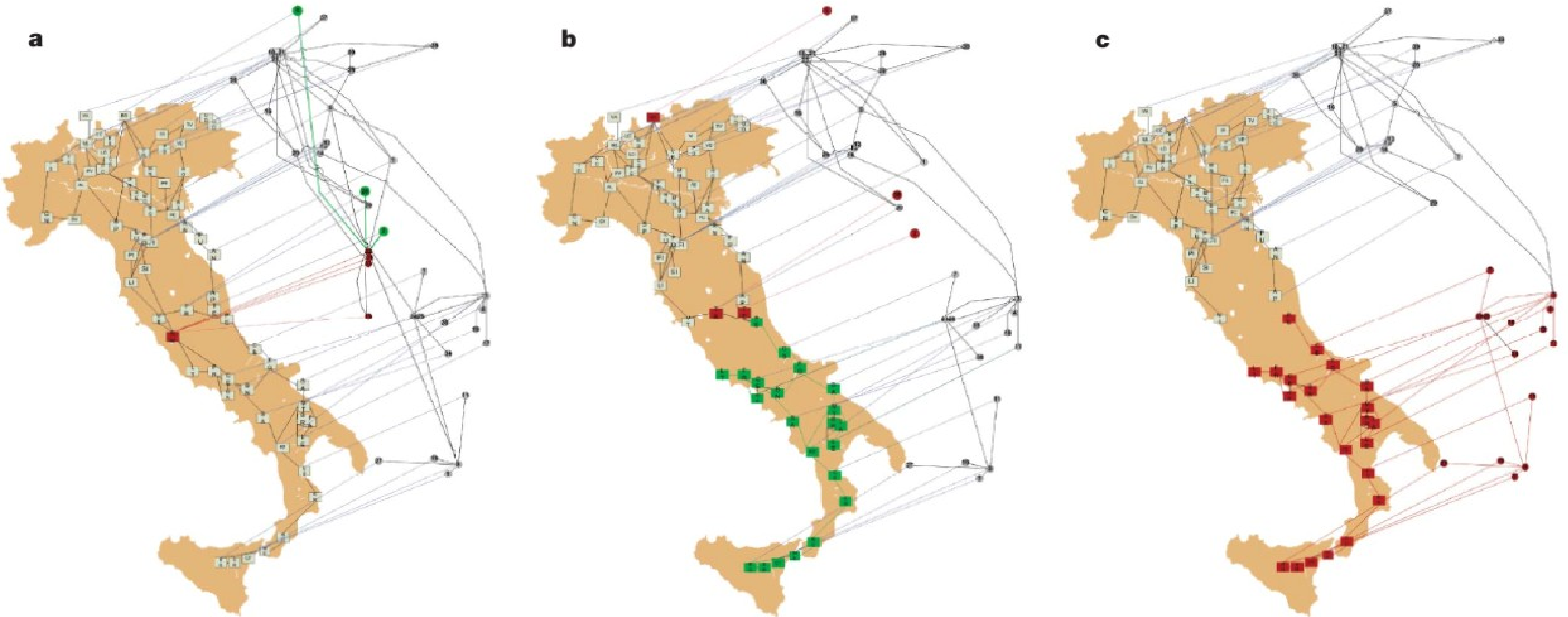
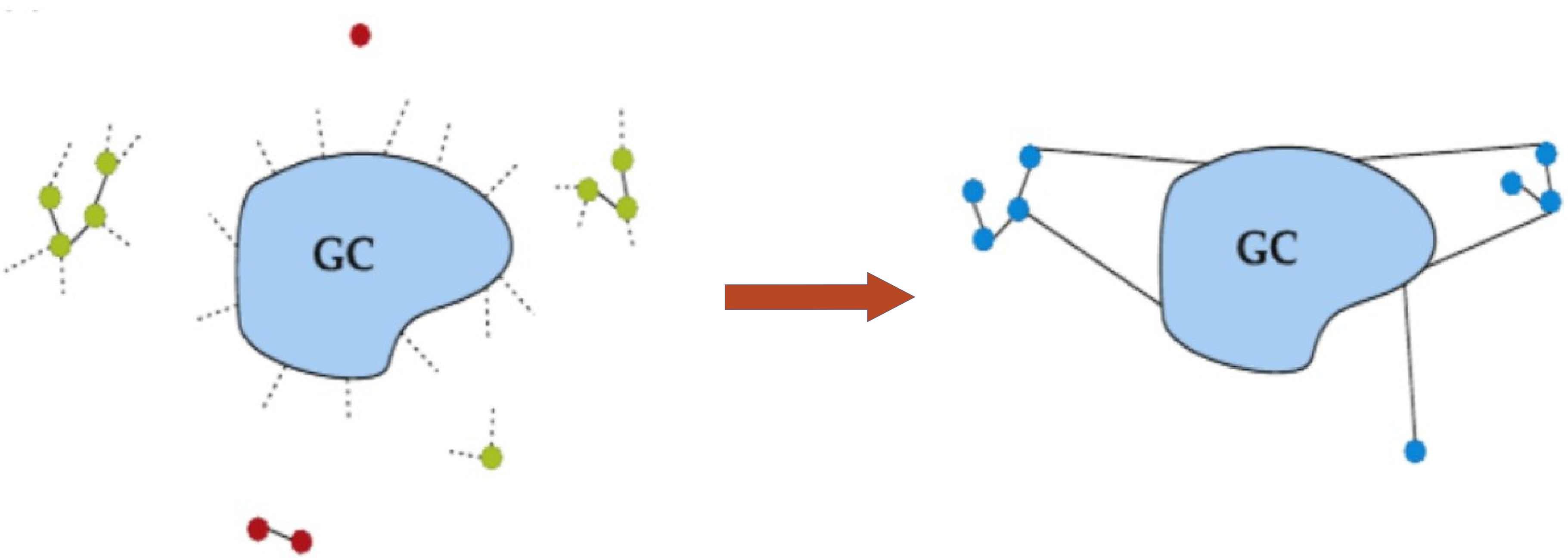


Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003[20]. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a**, One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network) at the next step are marked in green. **b**, Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c**, Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).

Estrategia: con probabilidad γ se reconecta a cada cluster finto con la CG.



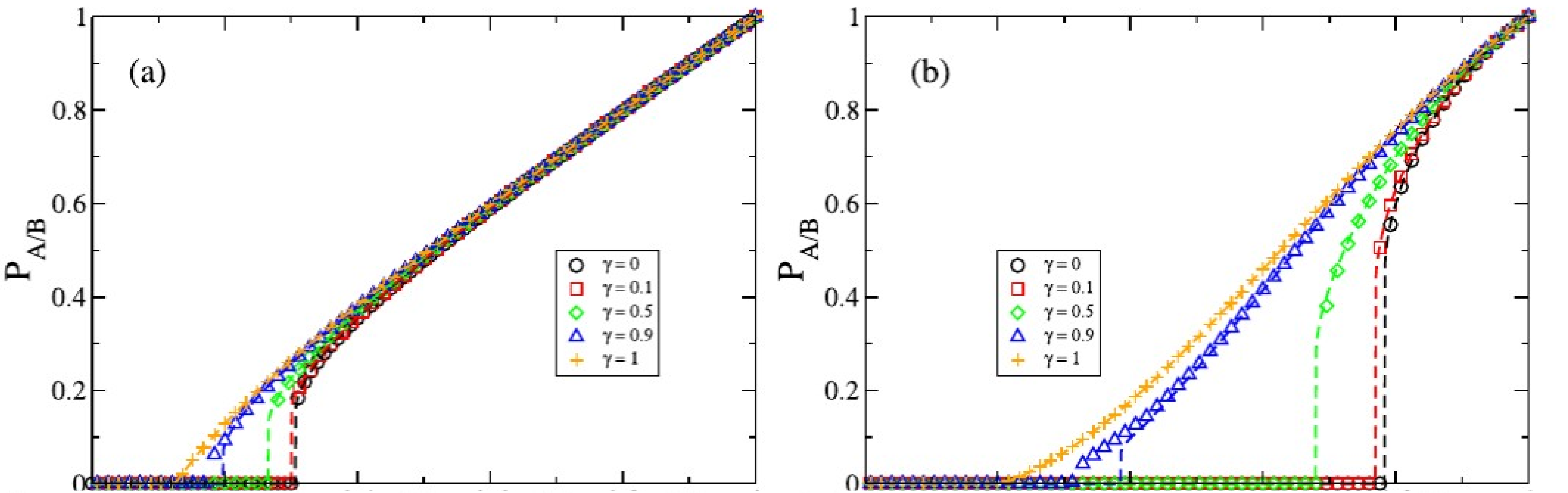
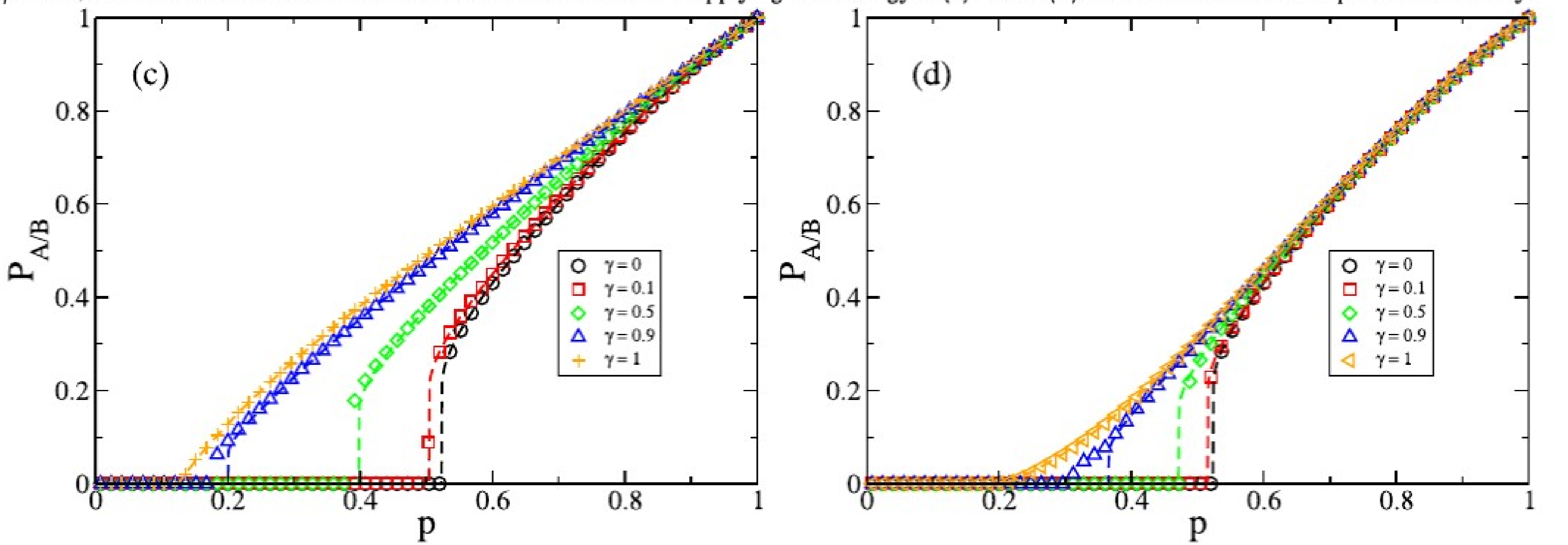


Fig. 2. P_∞ as a function of p for $\gamma = 0$ (◯), $\gamma = 0.1$ (□), $\gamma = 0.5$ (◇), $\gamma = 0.9$ (△) and $\gamma = 1$ (+) for (a) two ER, with $\langle k \rangle = 8$ (b) two SF with $\lambda = 2.5$ and $\beta = 20$, and the combination of the two kind of network ER and SF applying the strategy in (c) SF and (d) ER. The dashed curves represent the theory.



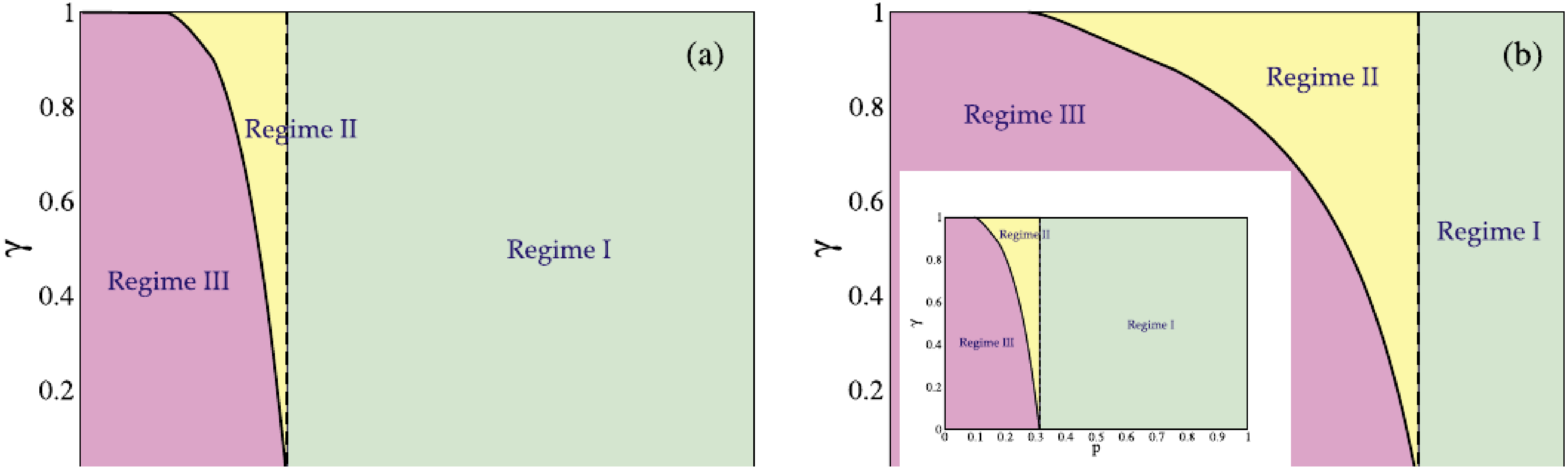
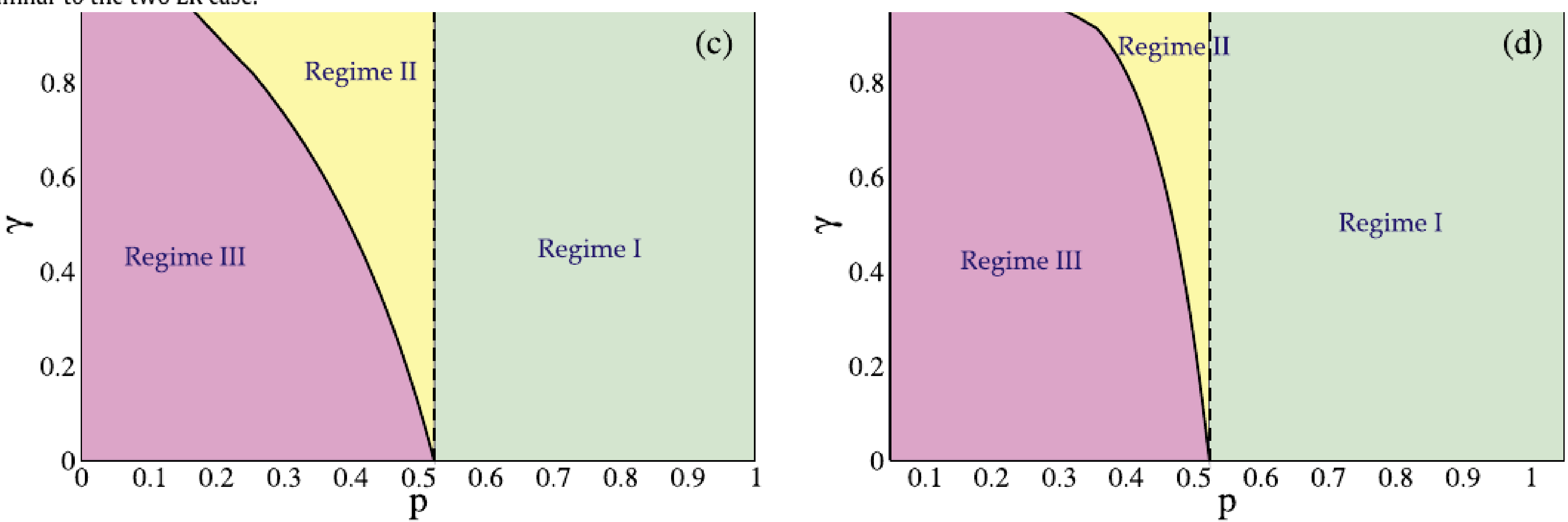


Fig. 4. Phase diagram in the plane γ–p from the theory for (a) two ER with $\langle k \rangle = 8$, (b) two SF with $\langle k \rangle = 3.08$ and the combination of and ER and SF networks applying the strategy in (c) SF and (d) ER. The continuous curve represents the value of γ_c below which the system is completely destroyed and the dashed line represents p_c for the case $\gamma = 0$. In the inset of (b) we plot the case for two SF, but with $k_{\min} = 5$ ($\langle k \rangle = 8$). Notice that the plot is very similar to the two ER case.



NOI: Number of Iterations

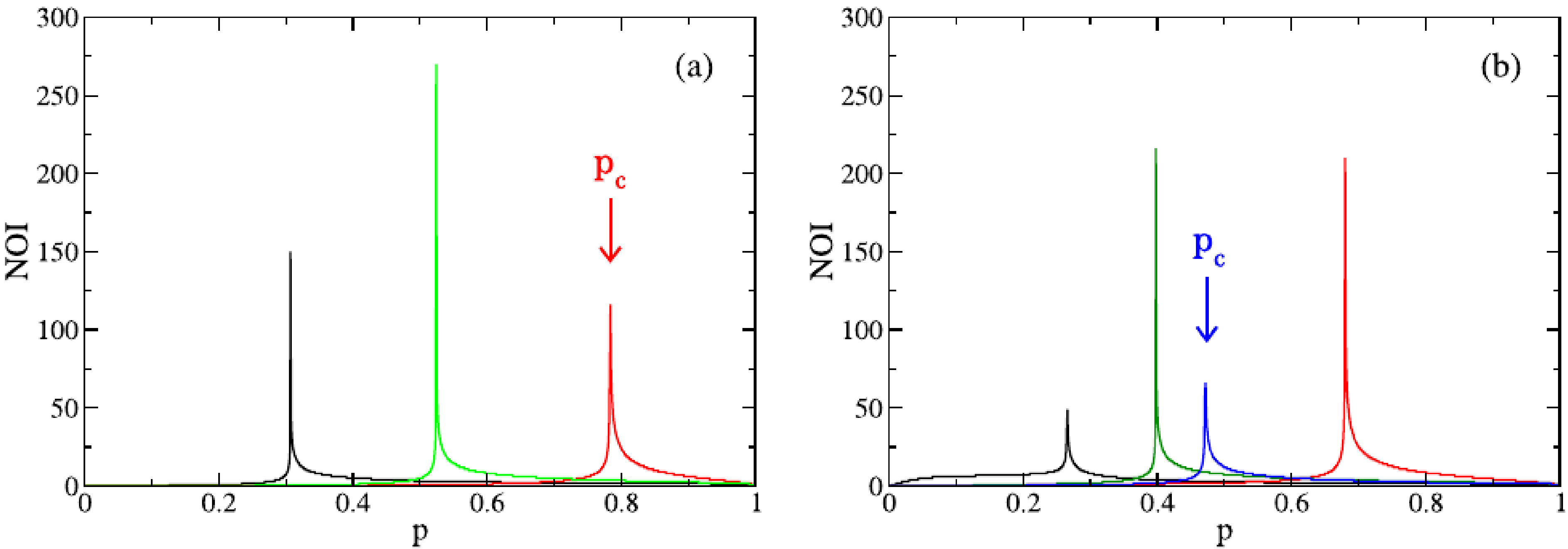


Fig. 3. NOI as a function of p from the theory for (a) $\gamma = 0$ and (b) $\gamma = 0.5$. Both cases are for two ER (black), two SF (red), ER–SF (green) and SF–ER (blue), always applying the strategy in B ($A - B$). Notice that in (a) the green curve represents the two cases ER–SF and SF–ER because without strategy the cascading failure is the same.

$$f = \mu(1 - G_1(1 - f))$$

$$P_\infty(\mu) = \mu(1 - G_0(1 - f))$$

$$\mu_A(n) = \mu_A(n-1)\left[1 - \frac{S_B(n-1)}{P_\infty^A(n-1)}\right]$$

$$S_A(n) = P_\infty^B(n-1) - P_\infty^A(n)$$

$$\mu_B(n) = \mu_B(n-1)\left[1 - \frac{S_A(n)}{P_\infty^B(n-1)}\right]$$

$$S_B(n) = P_\infty^A(n) - P_\infty^B(n)$$

Ecuaciones para la cascada sin estrategia

$$\mu_A(n) = \mu_A(n-1)\left[1 - \frac{(1-\gamma)S_B(n-1)}{P_\infty^A(n-1)}\right]$$

$$P_\infty^B(n) = P_\infty^B(n) + \gamma S_B(n)$$

Se suman a las anteriores con la estrategia

Conclusiones

- Desarrollamos una estrategia para evitar la completa destrucción de un sistema compuesto por dos redes interdependientes

- Aplicamos la estrategia solo en una de las redes y con probabilidad γ reconectamos a cada cluster finito con la CG

- Encontramos que el sistema se vuelve más robusto frente a la cascada al aumentar γ

- Es más efectiva la estrategia cuando se aplica en la red cuya conectividad media es más baja

- El problema se resolvió analíticamente usando la teoría de percolación

¡Muchas Gracias!