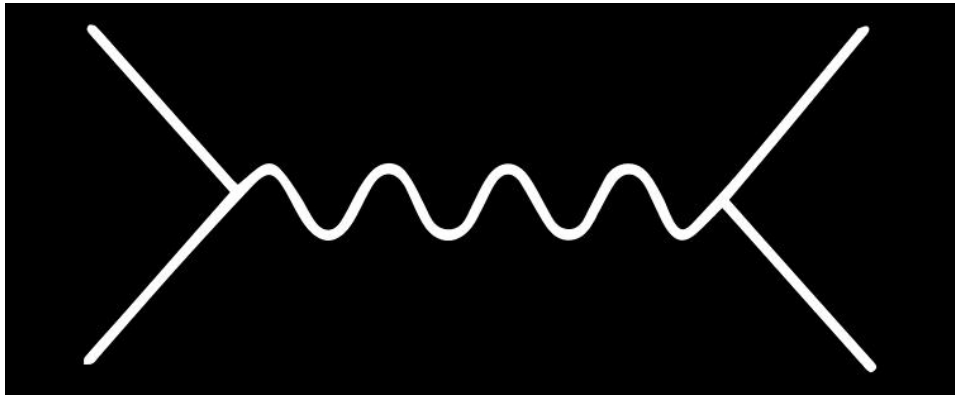


2da Ley de Newton y Diagramas de Feynman

El oscilador no lineal

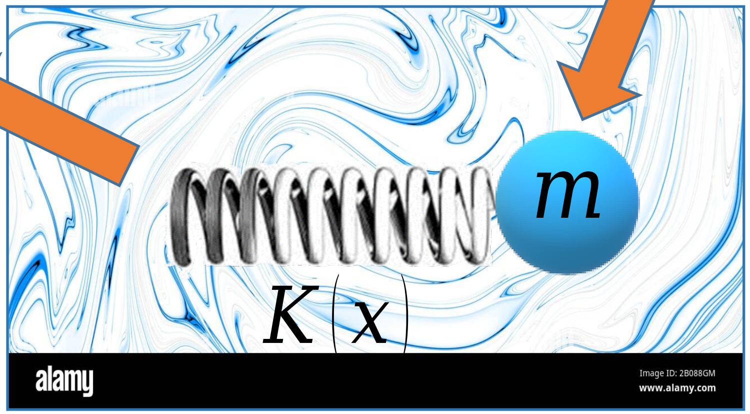
2da Ley de Newton

$$m \frac{d^2}{dt^2} x = \sum F_i$$

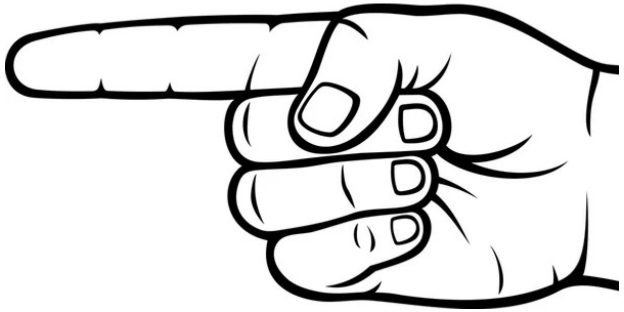
Oscilador no lineal forzado y amortiguado

$$-\gamma m \frac{d}{dt} x$$

$$-kx - \kappa x^3$$



$$F(t)$$

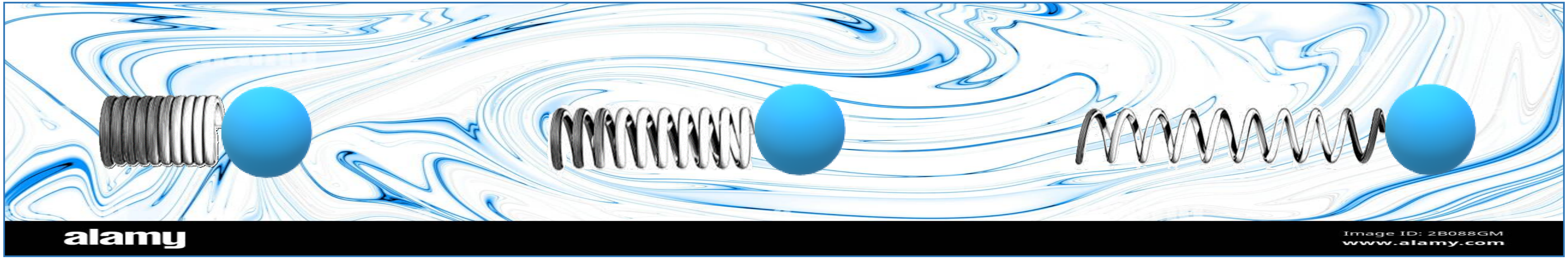


$$m \frac{d^2}{dt^2} x = -\gamma m \frac{d}{dt} x - kx - \kappa x^3 + F(t)$$

Oscilador armónico forzado y amortiguado

$$\frac{d^2}{dt^2}x = -\gamma \frac{d}{dt}x - \omega_0^2 x + f(t)$$

$$a\sin(\Omega t)$$



Función respuesta ó Función de Green

$$\left(\frac{d^2}{dt^2}+\gamma\frac{d}{dt}+\omega_0^2\right)x=f(t)$$

$$\left(-\omega^2-i\omega\gamma+\omega_0^2\right)\hat{x}(\omega)=\hat{f}(\omega)$$

$$\hat{x}(\omega)=G_0(\omega)\hat{f}(\omega)$$

$$\frac{\delta\hat{x}(\omega)}{\delta\hat{f}(\omega)}=G_0(\omega)=\frac{1}{\left(-\omega^2-i\omega\gamma+\omega_0^2\right)}$$

Propagador libre de Feynman

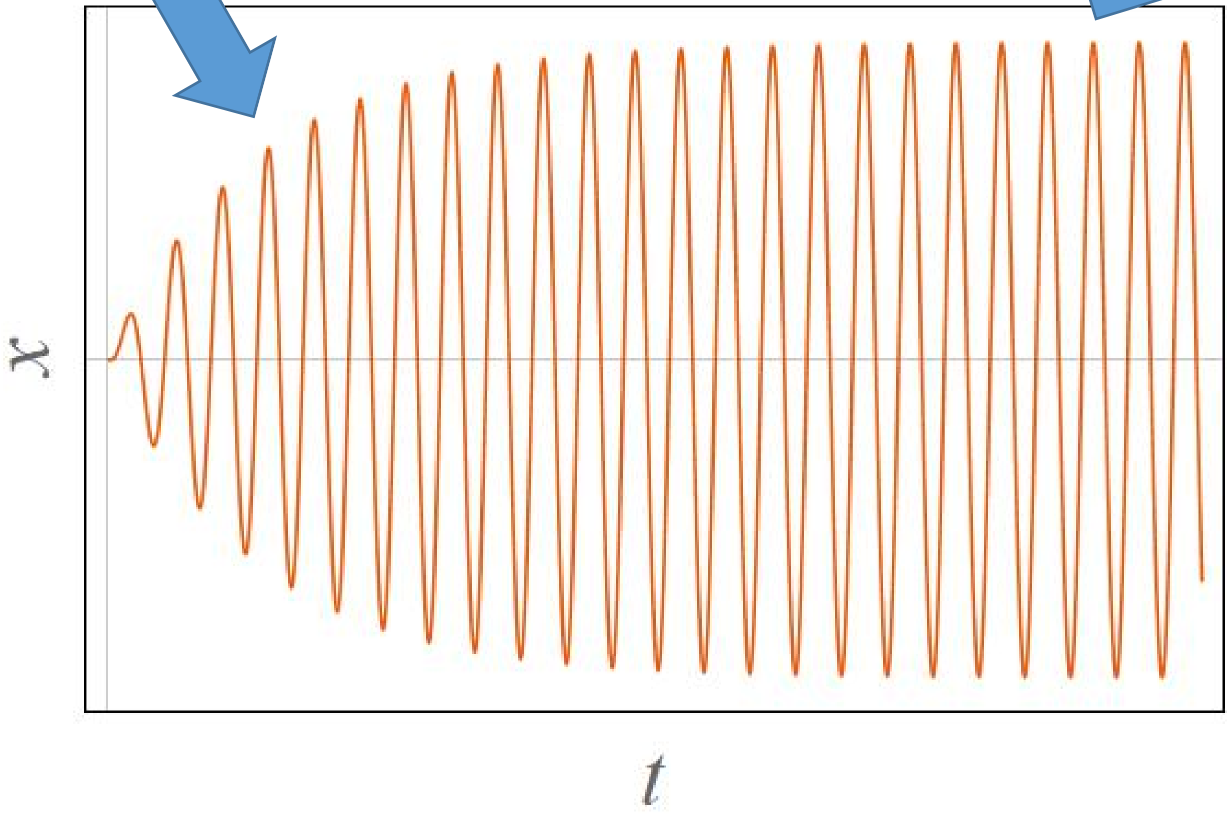
Solución al oscilador armónico forzado y amortiguado

Estado transitorio

$$x(t)=A\,sin(\Omega t)+B\,cos(\Omega t)+C(t)e^{-\gamma t/2}$$

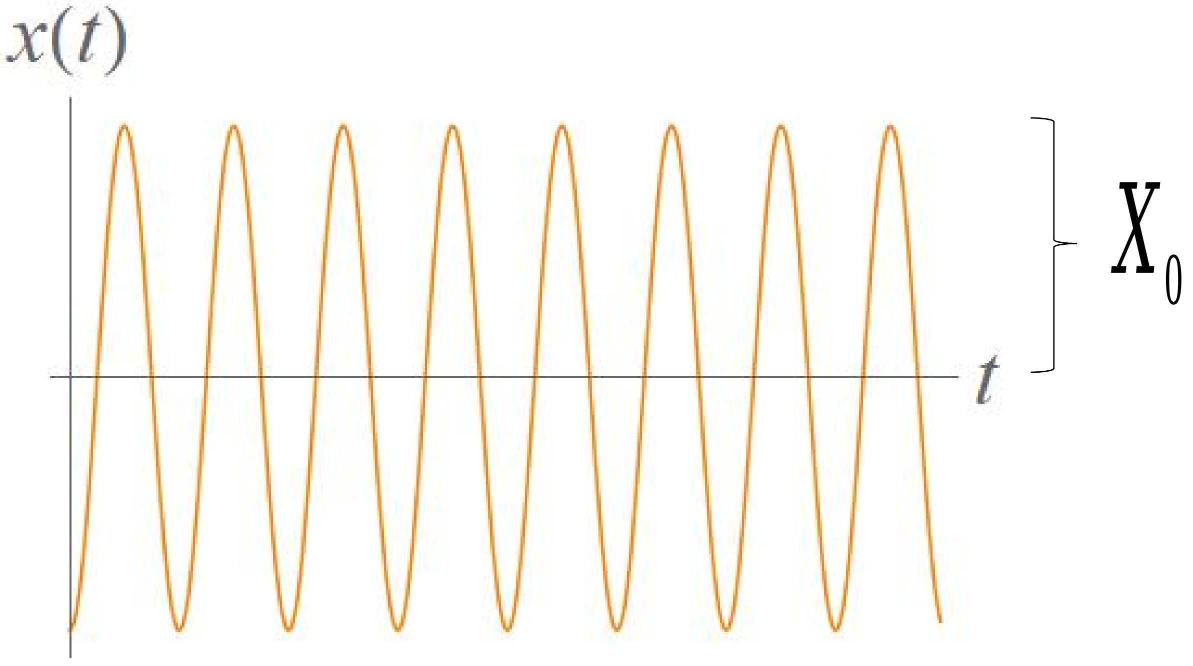
Estado estacionario

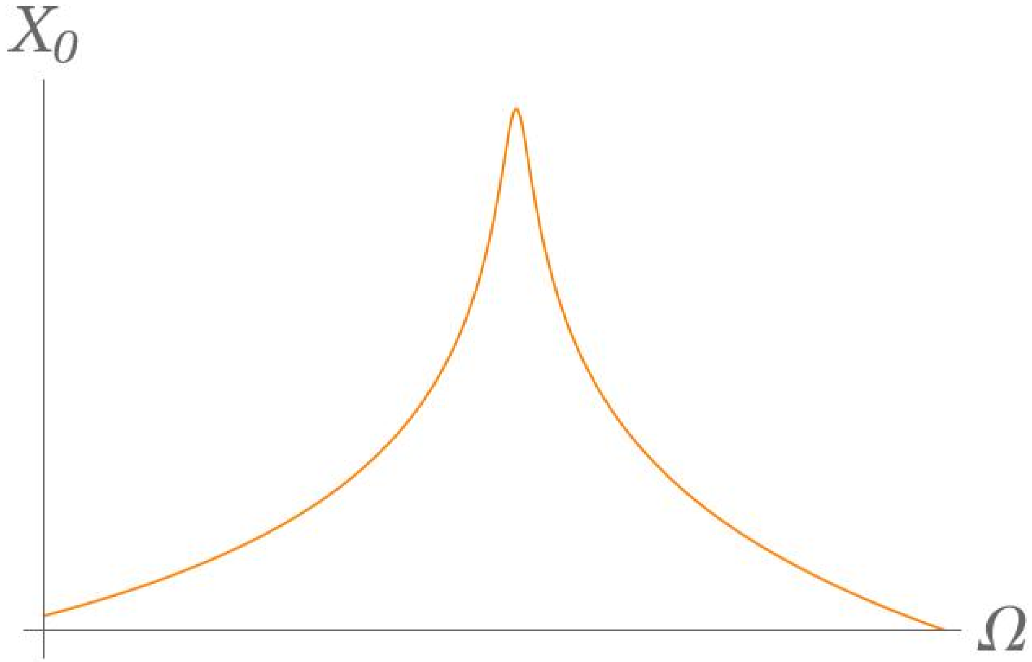
$$x(t)=X_0\,sin(\Omega t+\phi)$$



Solución al oscilador armónico forzado y amortiguado

$$X_0 = a\left|G_0(\Omega)\right|$$





$$\boxed{X_0 \text{ Máximo}} \iff \boxed{\Omega = \sqrt{\omega_0^2 - \gamma^2/2}}$$

$$\frac{d^2}{dt^2}x = -\gamma \frac{d}{dt}x - \omega_0^2 x - \lambda x^3 + f(t)$$

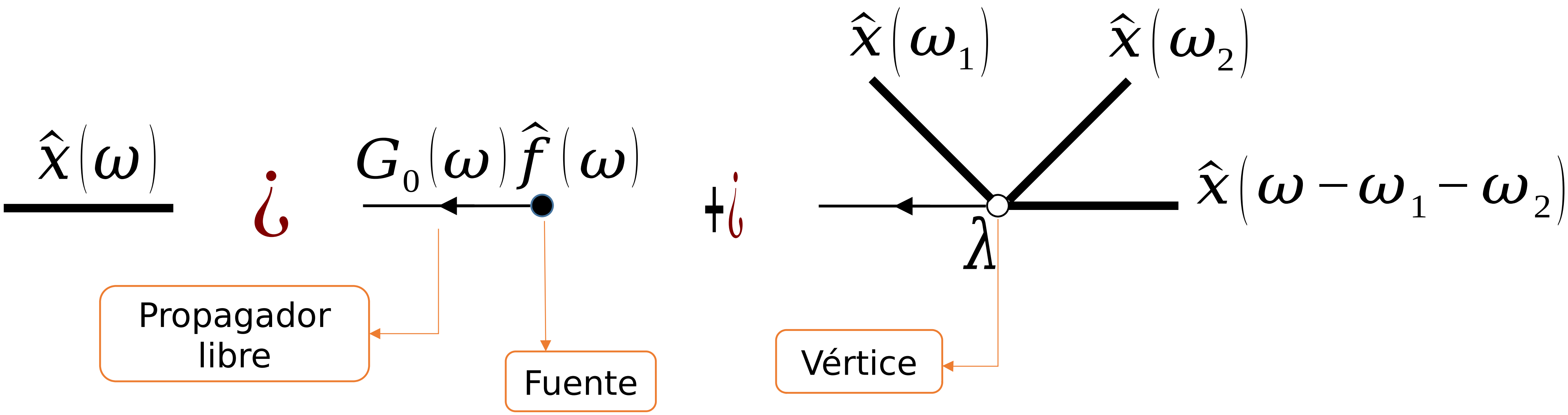
$$\left(-\omega^2 - i\omega\gamma + \omega_0^2\right)\hat{x}(\omega) = \hat{f}(\omega) - \lambda \mathscr{F}\left[x^3\right](\omega)$$

$$\hat{x}(\omega) = G_0(\omega)\hat{f}(\omega) - G_0(\omega)\lambda \mathscr{F}\left[x^3\right](\omega)$$

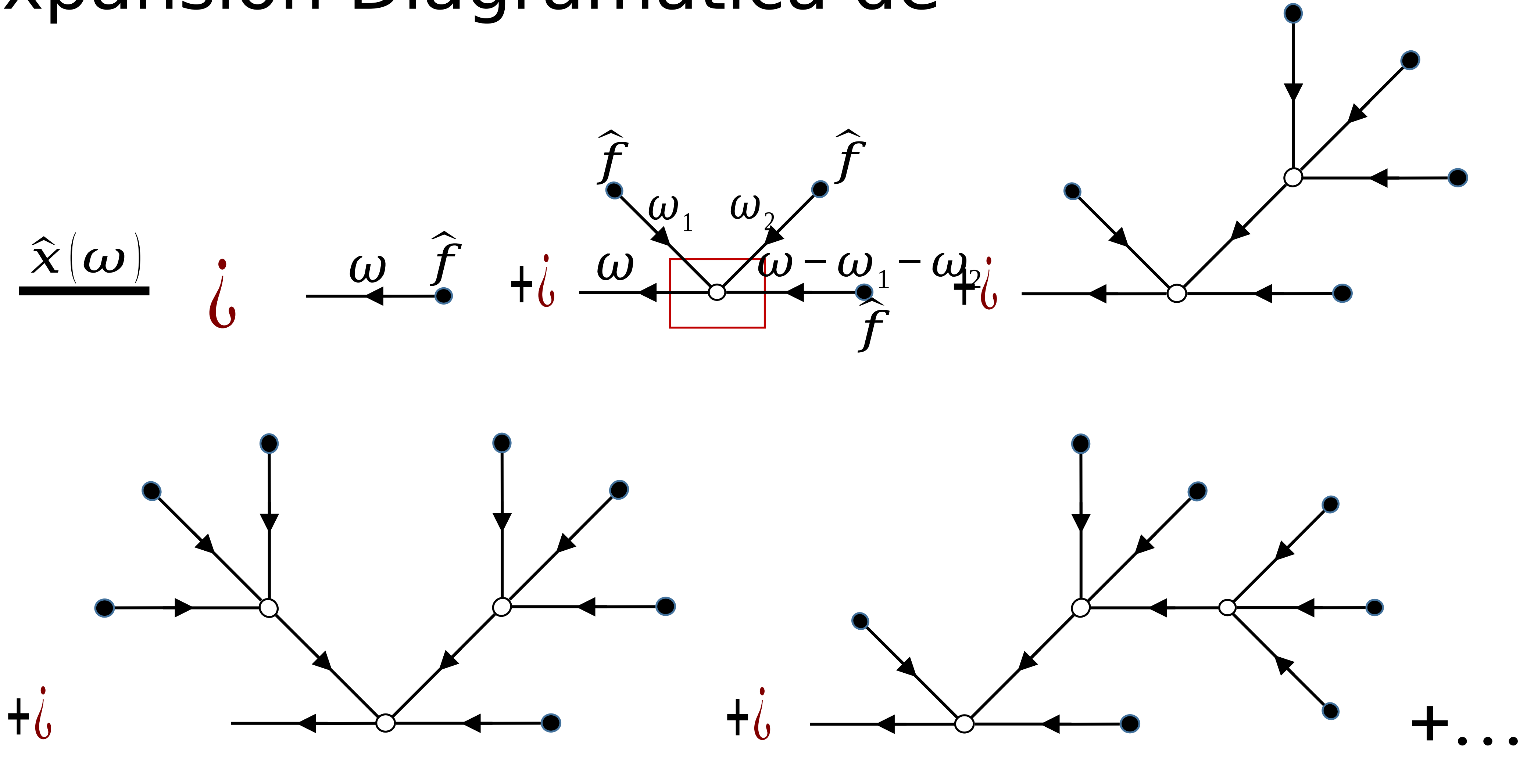
$$\hat{x}(\omega) = G_0(\omega)\hat{f}(\omega) - G_0(\omega)\lambda \int \hat{x}(\omega_1)\hat{x}(\omega_2)\hat{x}(\omega - \omega_1 - \omega_2)\, d\omega_1\, d\omega_2$$

Representación Diagramática

$$\hat{x}(\omega) = G_0(\omega)\hat{f}(\omega) + G_0(\omega)\lambda\int \hat{x}(\omega_1)\hat{x}(\omega_2)\hat{x}(\omega-\omega_1-\omega_2)\,d\omega_1 d\omega_2$$

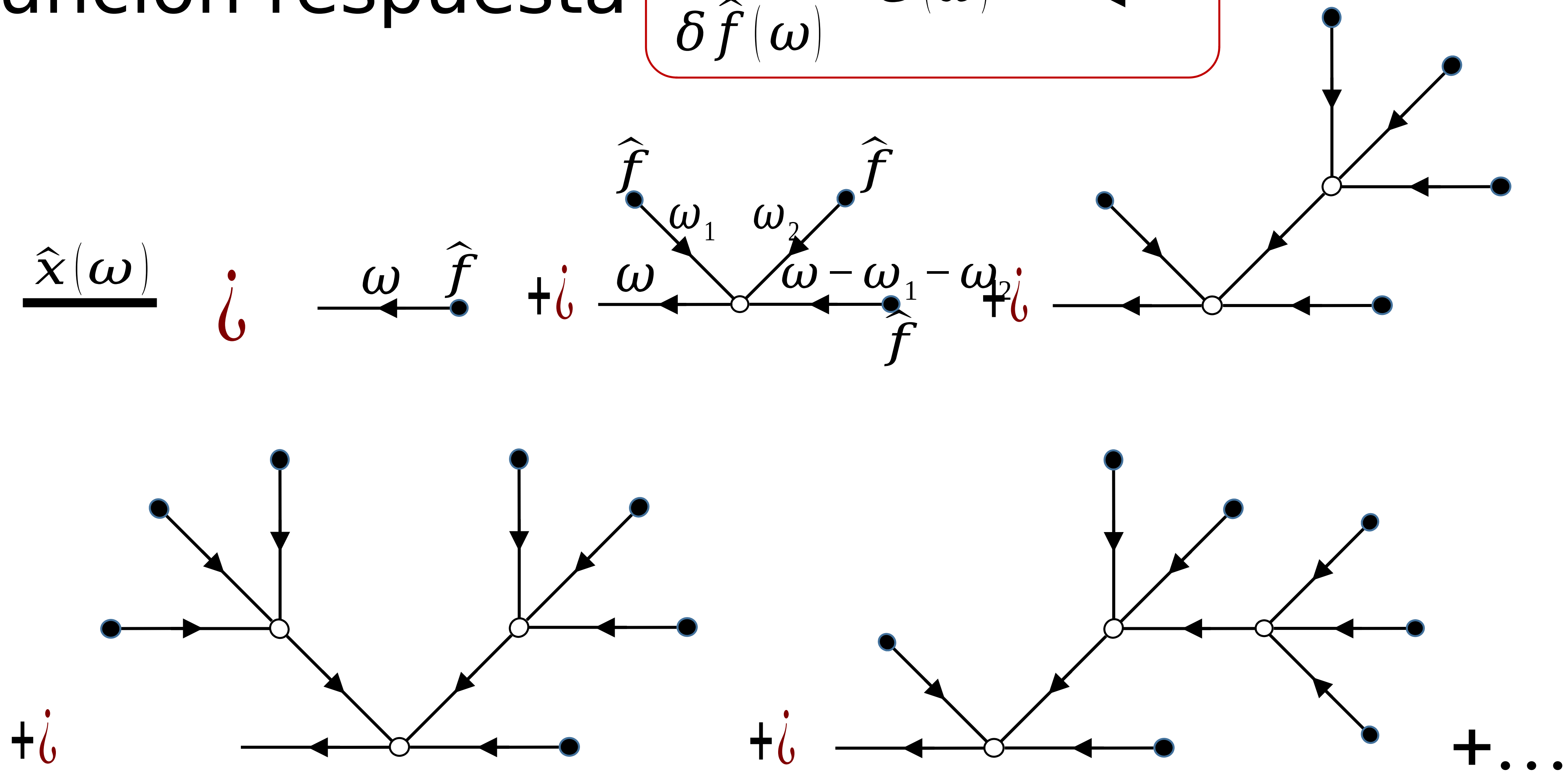


Expansión Diagramática de



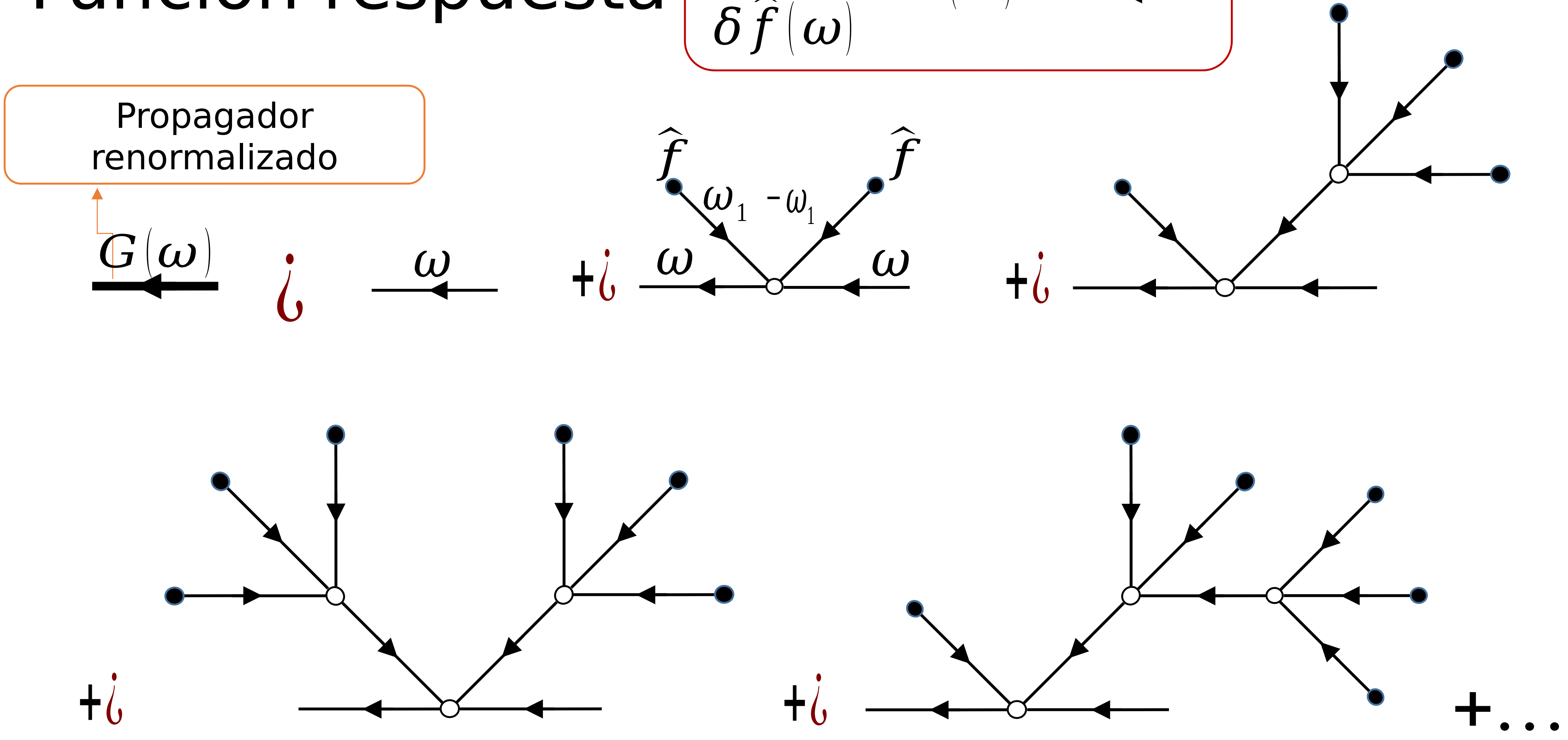
Función respuesta

$$\frac{\delta \hat{x}(\omega)}{\delta \hat{f}(\omega)} = G(\omega)$$

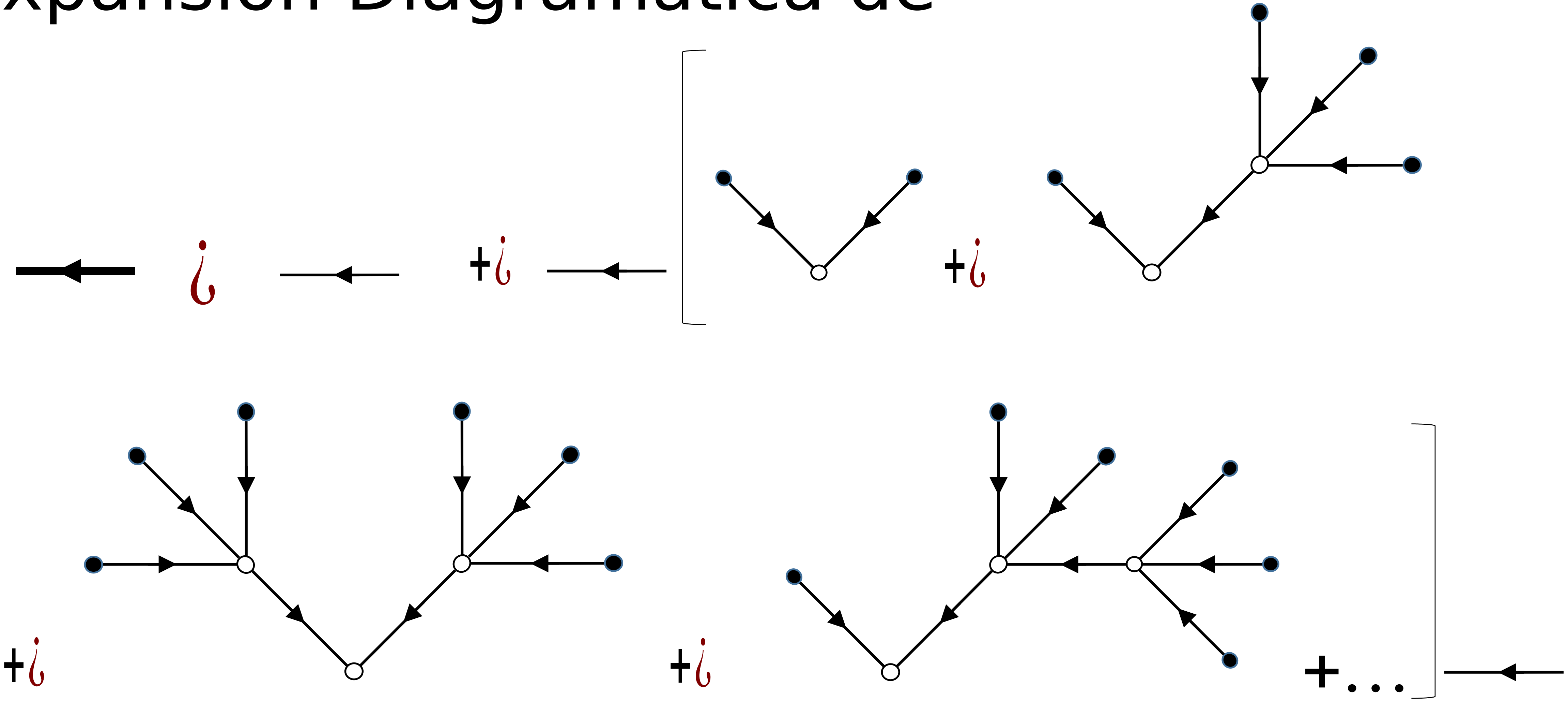


Función respuesta

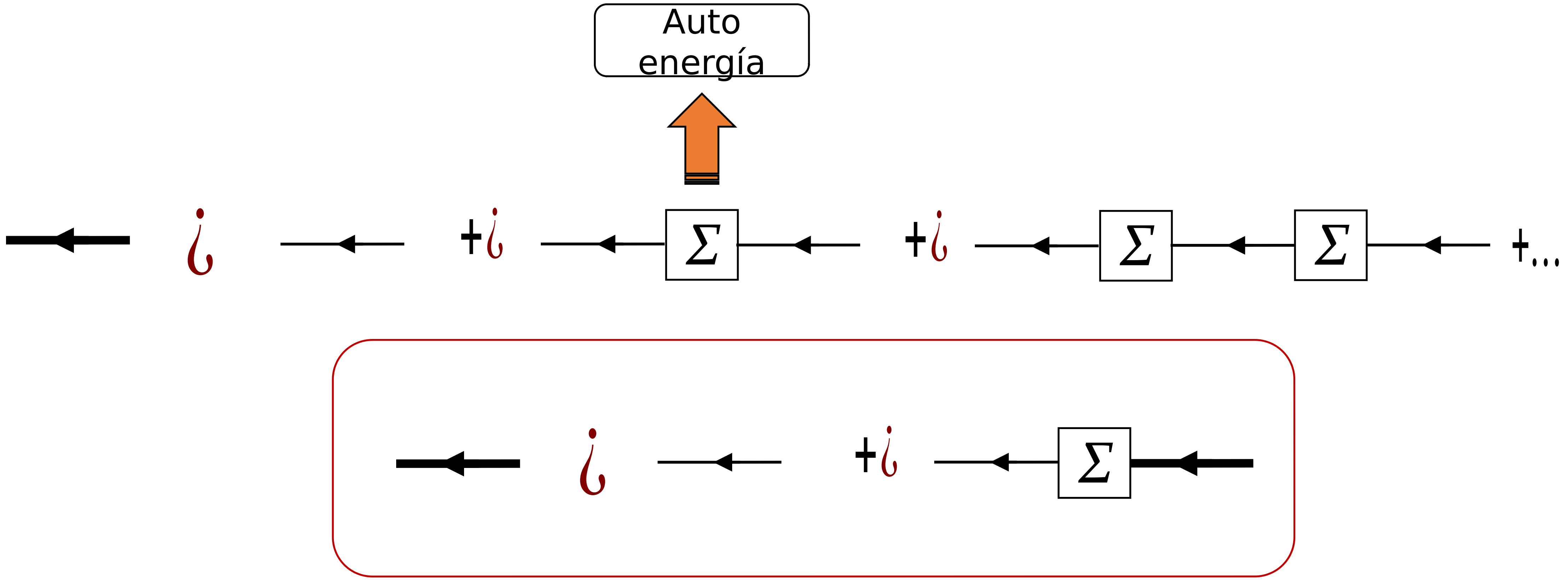
$$\frac{\delta \hat{x}(\omega)}{\delta \hat{f}(\omega)} = G(\omega)$$



Expansión Diagramática de



Ecuación de Dyson



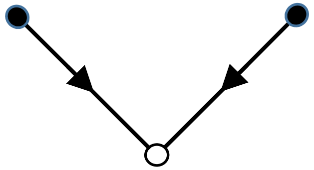
Ecuación de Dyson

$$G(\omega) = G_0(\omega) + G_0(\omega)\,\Sigma\, G(\omega)$$

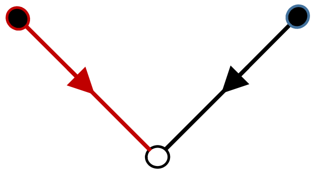
$$G^{-1}(\omega) = G_0^{-1}(\omega) - \Sigma$$

$$\boxed{G^{-1}(\omega) = -\omega^2 - i\omega\gamma + \omega_0^2 - \Sigma}$$

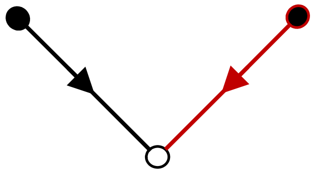
$\mathcal{O}(\lambda)$

$\Sigma =$ ¿  ¿ $-3\lambda \int d\omega_1 G_0(\omega_1)\hat{f}(\omega_1) G_0(-\omega_1)\hat{f}(-\omega_1)$

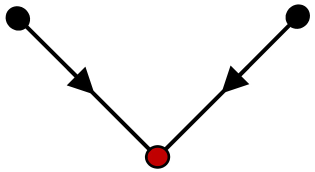
$\mathcal{O}(\lambda)$

$\Sigma =$ ¿  ¿ $-3\lambda \int d\omega_1 G_0(\omega_1)\hat{f}(\omega_1) G_0(-\omega_1)\hat{f}(-\omega_1)$

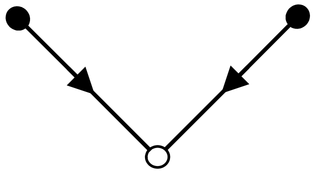
$\mathcal{O}(\lambda)$

$\Sigma =$ ¿  ¿ $-3\lambda \int d\omega_1 G_0(\omega_1)\hat{f}(\omega_1) G_0(-\omega_1)\hat{f}(-\omega_1)$

$\mathcal{O}(\lambda)$

$\Sigma = ¿$  $¿ -3\lambda \int d\omega_1 G_0(\omega_1)\hat{f}(\omega_1) G_0(-\omega_1)\hat{f}(-\omega_1)$

$\mathcal{O}(\lambda)$

$\Sigma = ¿$ 

$$¿ -3\lambda \int d\omega_1 G_0(\omega_1) \hat{f}(\omega_1) G_0(-\omega_1) \hat{f}(-\omega_1)$$

$$¿ -3\lambda \int d\omega_1 \left| \hat{x}_0(\omega_1) \right|^2$$

$$¿ -\frac{3}{2} \lambda a^2 \left| G_0(\Omega) \right|^2$$

$$\mathcal{O}(\lambda)$$

$$G^{-1}(\omega) = -\omega^2 - i\omega\gamma + \omega_0^2 - \Sigma$$

$$G^{-1}(\omega) = -\omega^2 - i\omega\gamma + \omega_0^2 + \frac{3}{2}\lambda a^2 \left|G_0(\Omega)\right|^2$$

$$G^{-1}(\omega) = -\omega^2 - i\omega\gamma + \omega_r^2$$

Solución al Oscilador no lineal forzado y amortiguado

$$x(t)=X_0(\omega_r,\gamma,\Omega)\,sin(\Omega t+\phi)$$

$$X_0=a|G(\Omega)|$$

$$\omega_r^2=\omega_0^2+\frac{3}{2}\lambda a^2|G_0(\Omega)|^2$$